Theory of Product Differentiation in the Presence of the Attraction Effect *

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Abstract

We apply the theoretical model of endogenous reference-dependence of Ok, Ortoleva and Riella (2011) to the theory of vertical product differentiation. We analyze the standard problem of a monopolist who offers a menu of alternatives to consumers of different types, but we allow for agents to exhibit a form of endogenous reference dependence like the attraction effect. We show that the presence of such biases might allow the monopolist to overcome some of the incentive compatibility constraints of the standard problem, leading the economy back towards the efficient equilibrium in which the monopolist extracts all the surplus. We then discuss welfare implications, showing that an increase in the fraction of customers who are subject to the attraction effect might not only increase the monopolist's profits and total welfare, but consumer's welfare as well.

JEL Classification: D11, L12, L15
Keywords: Attraction Effect, Reference-Dependence, Endogeneous Reference Formation, Vertical Product Differentiation.

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1 Introduction

In recent years a sizable literature in consumer psychology and in marketing has documented the presence and relevance of a specific form of systematic violation of rationality in choice: the so-called *attraction effect*, also known as the asymmetric dominance effect, or the decoy effect. In a nutshell, this effect could be described as the phenomenon according to which the introduction of an asymmetrically dominated alternative to a set might induce the agent to choose the dominating alternatives, possibly in violation of standard rationality. To illustrate, consider some decision maker who chooses between goods characterized by two attributes. Consider two alternatives of this kind, x and y, and suppose, as in Figure 1 (left), that x is better in one dimension, but y is better in the other one. Suppose that our agent chooses y when only x and y are available – our decision maker aggregates the two attributes and picks y. Suppose now that a third (decoy) alternative z is added to the set, as depicted in Figure 1 (right): indeed this alternative is *dominated* by x, but it is not dominated by y. The attraction effect phenomenon corresponds to the situation in which the introduction of the alternative z leads the agent to choose x when x, y, and z are available, in violation of rationality.

This phenomenon was first documented by Huber, Payne and Puto (1982), and it was later confirmed in a large number of studies in very different environments.[1] Importantly, it has been shown to play an important role also in field studies on actual supermarkets. In particular, Doyle et al. (1999) conducted experiments in a local grocery store in the UK. First, the authors recorded the weekly sales of the Brands X (the store's own brand, Spar (420 g) baked beans) and Y (Heinz (420 g) baked beans) in the grocery store under study, and observed that Brand X received 19% of the sales in a given week, and Y the rest, even though Brand X was cheaper. Doyle et al. then introduced a third Brand Z (Spar (220 g) baked beans) to the supermarket, which was identical to Brand X in all attributes (including the price) except that the size of Brand Z was visibly smaller. The idea here is that Brand Z was asymmetrically dominated; it was dominated by X but not by Y. In accordance with the attraction effect, the authors observed in the following week that the sales of Brand X had increased to 33% (while nobody had bought Brand Z).

Despite the large evidence in support of the importance of the attraction effect in the

[1]The attraction effect is demonstrated in the contexts of choice over political candidates (Pan, O'Curry and Pitts (1995)), choice over risky alternatives (Wedell (1991) and Herne (1997)), medical decision-making (Schwartz and Chapman (1999) and Redelmeier and Shafir (1995)), investment decisions (Schwarzkopf (2003)), job candidate evaluation (Highhouse (1996), Slaughter, Sinar and Highhouse (1999), and Slaughter (2007)), and contingent evaluation of environmental goods (Bateman, Munro and Poe (2008)). While most of the experimental findings in this area are through questionnaire studies, some authors have confirmed the attraction effect also through experiments with incentives (Simonson and Tversky (1992) and Herne (1999)).
In the psychological literature, it is argued that the attraction effect may be due to simplifying decision heuristics (Wedell (1991)), or due to one's need to justify his/her decisions (Simonson (1989), and Simonson and Nowlis (2000)), or due to the ambiguity of the information about the attributes of products (Ratneshwar, Shocker and Stewart (1987) and Mishra, Umesh and Stem (1993)), or due to the comparative evaluation of goods (Simonson and Tversky (1992) and Bhargava, Kim and Srivastava (2000)), or dynamic formation of preferences in a dominance-seeking manner (Ariely and Wallsten (1995)), or evolutionary pressures (Shafir, Waite and Smith (2002)). In the marketing literature see, inter alia, Burton and Zinkhan (1987), Lehmann and Pan (1994), Sivakumar and Cherian (1995), Sen (1998), Kivetz (1999), and Doyle, O'Connor, Reynolds and Bottomley (1999).

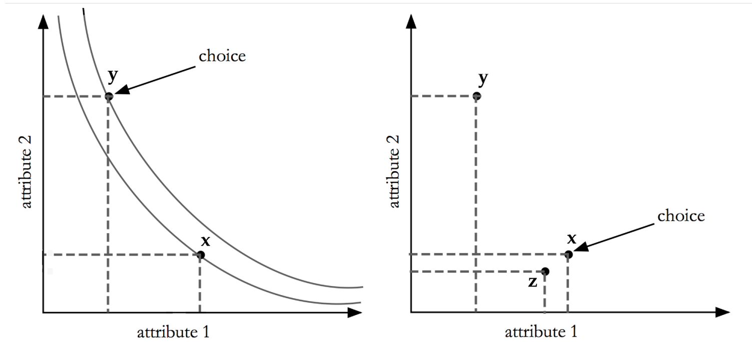

Figure 1. Illustration of the attraction effect

market, perhaps due to the lack of a suitable model of consumer's choice, to our knowledge the literature in economics contains no formal discussion of how such phenomenon might affect standard economic models: for example, whether a monopolist could exploit the attraction effect in the economy to extract more surplus; or if, and how, this would affect the bundles that she would offer. The goal of this paper is precisely to fill this gap: we study a standard model of vertical product differentiation, in which we assume that a fraction of the customers is potentially subject to the attraction effect.

We consider a standard model of vertical product differentiation, based on the one suggested by Mussa and Rosen (1978). A monopolist decides the (observable) price and quality of her goods, and she faces consumers who could be of different types, unobservable to her. In the standard model there are two types of consumers, Hight and Low, depending on their relative evaluation of price and quality. In our model, instead, there will be four types: for each type H and L, a fraction of customers is subject to the attraction effect. The monopolist knows the distribution of types in the population, and in particular she knows how many customers are subject to the attraction effect, but like in the standard model she does not observe the type of each consumer, and, therefore, she is unable to perfectly discriminate. Rather, like in the standard model, she will offer bundles of products targeting different types. Since a fraction of the consumers are subject to the attraction effect, the monopolist might choose to offer not only goods targeted at specific types, but also some *decoy* goods, meant to attract the sales towards some more profitable bundles. Since such decoy bundles will never be sold – they must be dominated to be 'decoy' – we need to specify the cost that the monopolist incurs in producing them: we use the parameter γ to indicate whether the monopolist needs to produce one decoy bundle for every customer she wishes to attract ($\gamma = 1$), or she doesn't need to produce it at all, since it's never sold ($\gamma = 0$), or any case in between ($\gamma \in (0, 1)$).

In order to develop our model, however, we need a way to model the behavior of consumers who are subject to the attraction effect. To this end, we use the reference-dependence choice model recently developed in Ok, Ortoleva and Riella (2011), which allows precisely for such case. According to this model, agents are endowed with a *utility function* U for each alternative, a *reference map* $\mathbf{r}$ which assigns to each set either an element of it, or declares that set to be free of reference effects, and a map Q which gives for every alternative its *attraction region*, that is, the set of options that alternative attracts the agent to. The choice procedure works as follows. For every set S, the reference maps specify if there is a reference point, or not, and if there is, which one it is. When there is no reference point, then the agent acts fully rationally: she chooses the elements that maximize her utility U in that set. If, however, the set S admits a reference point $r(S)$, then the agent instead only looks at the available options that also belong to the attraction region of the reference point: she only looks at the options in $S \cap Q(r(S))$. Amongst those, however, she is fully rational. For our implementation of the model, we assume that the utility function is identical to that of the original types, the attraction region of each element is the set of options that dominate it in both price and quality, and the reference map identifies a reference point if and only if there are dominated options in the set, the best of which acts as a reference point.

Our main findings are the following. First, we show that the monopolist will never want to exploit the attraction effect for consumers of the Low type: if she does, it is only to 'attract' customers of the high type. We then show that, as long as the cost of producing decoy options (γ) is not too high, or as long as the fraction of high types who are subject to the attraction effect is high enough, the monopolist will exploit the fact that a fraction of the high types is subject to the attraction effect. To this end, she will produce a decoy good, strategically placed to attract customers. In particular, we show that in the special case in which producing the decoy good is costless ($\gamma = 0$), and in which every high type is subject to the attraction effect, then the economy is back to efficiency: thanks to the attraction effect, the monopolist is able to perfectly segment the market, reach first best solution, and extract *all* the surplus.

Finally, we analyze the welfare effects of an increase on the proportion of customers who are subject to the attraction effect. We show that in this case the profit of the monopolist increases, as does the total welfare of the economy. Moreover, there are parameter ranges in which also the consumer surplus increases: while the low types are always indifferent, the high types who are not subject to the attraction effect are actually better off; the only customers who loose welfare are those who become subject to the attraction effect – but for many parameter values, the former effect dominates the latter, leading to the situation in which, in expectations, for some parameter values customers would be better off the more of them are bounded rational.

Our model fits in the growing literature that applies to industrial organization choice models which depart from standard rationality. We refer to Spiegler (2011) for an excellent survey. Although to the best of our knowledge there is no such models that studies the consequences of the attraction effect, there are studies that analyze product differentiation in the presence of other forms of departure from standard rationality. Amongst them, Esteban and Miyagawa (2006) and Esteban, Miiyagawa and Shum (2007) study the case in which consumers are characterized by Self-Control preferences *à la* Gul and Pesendorfer (2001). They show that if the high type's marginal value for the quality of goods raises with the temptation, then the monopolist is able to achieve perfect discrimination, overcoming the incentive problems – as it happens in our paper. As opposed to our results, however, they use very different departures from rationality, and their approach is based on menus, which makes them depart from the standard model in a way that we don't need.

The remainder of the paper is organized as follows. Section 2 describes the standard model of vertical product differentiation of Mussa and Rosen (1978). Section 3 describes the reference-dependent choice model of Ok, Ortoleva and Riella (2011). Section 4 presents our model, its solution, and the welfare implications. The proofs omitted from the main text appear on the appendix.

2 The standard model of vertical product differentiation: Mussa and Rosen (1978)

We begin our analysis by discussing the standard model of vertical product differentiation, developed in Mussa and Rosen (1978). This model will serve as a benchmark for the subsequent analysis.

Consider a monopolistic market for a single good, where there are two types of consumers with distinct taste parameters about the quality of the product. The two types are H (for high) and L (for low), and we assume that they are evenly distributed in the society. Types H and L evaluate the utility of one unit of the good of quality $q \geq 0$ at price $p \geq 0$ as

$$U_L(p,q) := \theta_L q - p \text{ and } U_H(p,q) := \theta_H q - p,$$

where $\theta_H > \theta_L > 0$. For concreteness, we work here with a particular production technology by confining ourselves to the case of quadratic cost functions. That is, we assume that the cost of producing a unit good of quality $q \geq 0$ is q^2. The problem of the monopolist is then to choose quality levels $q_H, q_L \geq 0$ and unit prices $p_H, p_L \geq 0$ in order to solve the following maximization problem:

$$\begin{aligned} \Pi^* = \max \quad & (p_L - q_L^2) + (p_H - q_H^2) \\ \text{such that} \quad & U_L(p_L, q_L) \geq 0, \\ & U_H(p_H, q_H) \geq 0, \\ & U_H(p_H, q_H) \geq U_H(p_L, q_L), \\ & U_L(p_L, q_L) \geq U_L(p_H, q_H). \end{aligned}$$

In what follows we work with the parametric restriction $\theta_L > \frac{\theta_H}{2}$. This condition guarantees the presence of a (unique) strictly positive solution for the monopolist's problem, simplifying our analysis and allowing us to avoid the treatment of less interesting corner solutions. This solution has the first and third constraints binding and the optimal quality choices of the monopolist are found as:

$$q_H^{MR} = \frac{\theta_H}{2} \quad \text{and} \quad q_L^{MR} = \theta_L - \frac{\theta_H}{2}.$$

(The product produced by the monopolist for the H types is shown by $\hat{H}$ in Figure 2, and that for L types as $\hat{L}$.) As it is common with this type of screening models, the solution is 'efficient at the top,' in the sense that q_H^{MR} is the efficient (socially optimal) level of quality. On the other hand, there is a downward distortion of the low valuation agent's quality with respect to the first-best outcome. In what follows, we shall refer to the menu $\{(p_H^{MR}, q_H^{MR}), (p_L^{MR}, q_L^{MR})\}$ as the MR solution.[2]

[2]It is easily computed from the constraints of the problem that

$$p_H^{MR} = \frac{\theta_H^2}{2} - (\theta_H - \theta_L)(\theta_L - \frac{\theta_H}{2}) \quad \text{and} \quad p_L^{MR} = \theta_L(\theta_L - \frac{\theta_H}{2}).$$

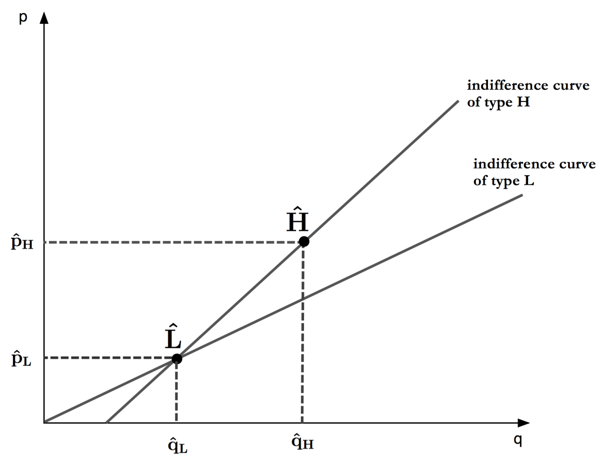

Figure 2. Solution of the standard vertical product differentiation model.

3 The Reference-Dependent Model of Ok, Ortoleva and Riella (2011)

The main departure from the standard model of this paper is to allow consumers to be subject to forms of endogenous reference-dependence such as the attraction effect. We model this using the model of Ok, Ortoleva and Riella (2011), which we now describe. Consider a fixed separable metric space X: we think of X as the universal set of all distinct choice alternatives. We define $\mathfrak{X}$ to be the collection of all compact subsets of X. An element S of $\mathfrak{X}$ is referred to as a *choice problem* or a *choice situation.* The model studies the choice behavior of an agent, and the following definition is basic.

Definition 1. A correspondence $c : \mathfrak{X} \rightrightarrows X$ is said to be a **choice correspondence** on $\mathfrak{X}$ if $\emptyset \neq c(S) \subseteq S$ for every $S \in \mathfrak{X}$.

We can now introduce the reference-dependent choice model. In what follows, we reserve the symbol $\diamondsuit$ for an arbitrary object that does not belong to X.

Definition 2. A **reference-dependent choice model** that represents a choice correspondence c is a triplet $\langle U, \mathbf{r}, Q\rangle$, where U is a continuous real function on X (utility function), $\mathbf{r} : \mathfrak{X} \to X \cup \{\diamondsuit\}$ is a map (reference map), and $Q : X \cup \{\diamondsuit\} \rightrightarrows X$ is a correspondence (attraction region) such that,

1. For any $S \in \mathfrak{X}$,
$$c(S) = \arg\max U(S \cap Q(\mathbf{r}(S))) \tag{1}$$
2. $\mathbf{r}$ is a reference-map: for any $S \in \mathfrak{X}$ we have $\mathbf{r}(S) \in S$ whenever $\mathbf{r}(S) \neq \diamondsuit$. And for any $x, y \in X$, $\mathbf{r}(\{x, y\}) = \diamond$;
3. $Q(\diamondsuit) = X$;

4. For any $S, T \in \mathfrak{X}$ with $\mathbf{r}(S) \in T \subseteq S$, and $\arg\max U(S \cap Q(\mathbf{r}(S))) \cap T \neq \emptyset$, we have

$$\arg\max U(T \cap Q(\mathbf{r}(T))) = \arg\max U(T \cap Q(\mathbf{r}(S))).$$

The interpretation is the following. Here U is interpreted as the utility function of the individual decision maker, *free of any referential considerations*. In particular, if the alternatives have various attributes that are relevant to the final choice – these attributes may be explicitly given, or may have a place in the mind of the agent – then U can be thought as aggregating the performance of all the attributes of any given alternative in a way that represents the preferences of the agent.

In turn, $\mathbf{r}$ serves as the reference map that tells us which alternative is viewed by the agent as the reference for a given choice situation: given any choice problem S in $\mathfrak{X}$, a reference map $\mathbf{r}$ on $\mathfrak{X}$ either identifies an element $\mathbf{r}(S)$ of S, which we understand as acting as a reference point when solving this problem; or it declares that no element in S qualifies to be a reference for the choice problem – we denote this situation by $\mathbf{r}(S) = \diamondsuit$, where the symbol $\diamondsuit$ represents the idea of 'empty.' Under this interpretation, the requirement that for any $x, y \in X$ we have $\mathbf{r}(\{x, y\}) = \diamond$ is related to the fact that the notion of "reference alternative" that we intend to capture is not related to, say, the status quo bias phenomenon. The latter notion would necessitate a default option to be thought of as a "reference" in dichotomous choice problems as well. As we mentioned in the introduction, the focus of the model of Ok, Ortoleva, Riella (2011) is on the notion of "reference" alternatives that are not desirable in themselves, but rather, affect the comparative desirability of other alternatives. Thus, the reference notion becomes meaningful in the present setup only when there are at least two alternatives in the choice situation at hand, in addition to the alternative designated as the reference point.

The interpretation of the correspondence Q is more subtle. For any $\omega \in X \cup \{\diamondsuit\}$, we interpret the set $Q(\omega)$ as telling us which alternatives in the universal set X look "better" to the agent *when compared to* ω – it may thus make sense to call $Q(\omega)$ the *attraction region* of ω. (For instance, if the agent deems a number of attributes of the alternatives as relevant for her choice, then $Q(\omega)$ may be thought of as the set of all alternatives that dominate ω with respect to all attributes.) Accordingly, we have $Q(\diamondsuit) = X$ (condition 3) – "nothing" does not attract the agent's attention to any particular set of alternatives, so every one of them belongs to its attraction region.

Given these definitions, take any choice problem $S \in \mathfrak{X}$. The agent either evaluates this problem in a reference-independent manner, or she identifies a reference point in S and uses this point to finalize her choice. In the former case, $\mathbf{r}(S) = \diamondsuit$, so, by condition (3), $Q(\mathbf{r}(S)) = X$, which means that, in this case (1) reads

$$c(S) = \arg\max U(S),$$

in concert with the standard theory of rational choice.[3] That is, when there is no reference point ($\mathbf{r}(S) = \diamondsuit$), the agent behavior coincides with that of a standard agent. On the other hand, when there is a reference point ($\mathbf{r}(S) \neq \diamondsuit$), then the agent is mentally "attracted" to the elements of S that belong to $Q(\mathbf{r}(S))$. It is "as if" she faces the *mental constraint* that

[3]In fact, the standard theory is thus captured by $\langle U, \mathbf{r}, Q \rangle$ upon setting $\mathbf{r}(S) = \diamondsuit$ for all $S \in \mathfrak{X}$.

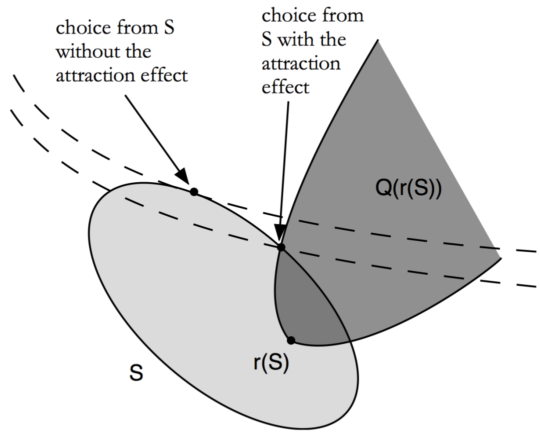

Figure 3. The Reference-Dependent Choice model of Ok, Ortoleva, Riella (2011).

her choices from S must belong to $Q(\mathbf{r}(S))$ – and she disregards any option that does not belong to this set. As illustrated in Figure 3, however, within this constraint the agent acts fully rationally, and solves her problem upon the maximization of U, that is,

$$c(S) = \arg\max U(S \cap Q(\mathbf{r}(S))).$$

Finally, the rationale for Condition (4) is the following. Without this condition, in general such a model would not at all restrict how the reference points of an agent relate to each other across varying choice situations. To illustrate, take a choice correspondence c on $\mathfrak{X}$ that is represented by such a model $\langle U, \mathbf{r}, Q \rangle$, and suppose that S is a choice problem in $\mathfrak{X}$ such that

$$\mathbf{r}(S) \neq \diamondsuit \text{ and } x \in c(S). \tag{2}$$

Then, x need not be a utility-maximizing alternative in S; it rather maximizes U over the subset of S that consists of all alternatives toward which the reference alternative $\mathbf{r}(S)$ "attracts" the agent (i.e. over $S \cap Q(\mathbf{r}(S))$). Now consider another choice problem $T \in \mathfrak{X}$ such that

$$\{x, \mathbf{r}(S)\} \subseteq T \subseteq S. \tag{3}$$

The model $\langle U, \mathbf{r}, Q \rangle$ does not put any restrictions on what would the reference point in T be, and hence, on how the choice from T would relate to that from the larger set S. It may be that a new alternative $\mathbf{r}(T)$ now acts as a reference in T and an alternative y – whose utility may be significantly below $U(x)$ – is thus chosen by the agent due to this (i.e., because $y \in Q(\mathbf{r}(T))$ and $x \notin Q(\mathbf{r}(T))$). Put differently, the arbitrariness of the $\mathbf{r}$ function allows for rather wild violations of the Weak Axiom of Revealed Preference (WARP), thereby taking away significantly from the predictive strength of the model $\langle U, \mathbf{r}, Q \rangle$.

To avoid this, Condition (4) imposes some restraints on models of the form $\langle U, \mathbf{r}, Q \rangle$ that relate the references and choices across nested choice problems. It simply requires that, in these cases, even though we might have $\mathbf{r}(T) \neq \mathbf{r}(S)$, the choice from the set T is compatible with $\mathbf{r}(S) = \mathbf{r}(T)$, i.e. the choice from T would be identical if this held.

4 Vertical Product Differentiation with the Attraction Effect

4.1 General Discussion

We now apply the reference-dependent choice model described in the previous section to develop a model of vertical product differentiation, similar to the one described in Section 2, but in which at least a fraction of the consumers might be subject to the attraction effect. To simplify the exposition, and avoid the consideration of certain trivial cases, we shall carry out the condition from the MR model which guarantees an interior solution: we assume in what follows that $\theta_L > \frac{\theta_H}{2}$.

Suppose that some consumers in the market are subject to the attraction effect – we refer to these consumers as type A_H or A_L, depending on the fact that they are originally type H or L. Since we allow for the contemporaneous presence of both consumers who are and consumers who are not subject to the attraction effect for each original type of consumer, we therefore have four types of consumers in this market: H, L, A_H, and A_L. Types H and L are modeled as in the standard case, while types A_H and A_L will be modeled following the model described in Section 3.[4] We are going to make a small modification to the setup in the MR model. Now we are going to define X to be $\mathbb{R}^2_+ \times \{0,1\}$. The idea is that the alternatives have a third binary attribute on top of their prices and qualities. The main reason we are introducing this third attribute is to avoid some technicalities in the statement of the monopolist's problem when some agents suffer from the attraction effect. As we have said, the choice behavior of type A_i consumers is modeled by means of a reference dependent choice model $\langle U_{A_i}, \mathbf{r}_i, Q\rangle$ on the set $\mathfrak{X}$ of all compact subsets of $\mathbb{R}^2_+ \times \{0,1\}$ as follows. First, we posit that these consumers maintain the utility function that they had in their original type, that is, $U_{A_H} = U_H$ and $U_{A_L} = U_L$. Second, we recall that a plausible interpretation of the attraction region $Q(\omega)$ of a choice alternative ω is as the set of all alternatives that dominate ω with respect to each attribute relevant for choice. This interpretation fits particularly well in the present setup. For a given alternative (p, q, b) we define the attraction region of (p, q, b) by $Q(p, q, b) := \{(s, t, v) \in \mathbb{R}^2_+ \times \{0, 1\}\} : s \leq p,\, t \geq q$ and either both inequalities are strict or $v = b\} \cup \{(0, 0, 0)\}$. As we have discussed before, the main reason we have introduced the third attribute to the setup now is to avoid some technicalities in the statement of the monopolist's problem. If we had not done that, the maximum would not necessarily be attained in that problem. Notice that we also assume that the possibility of buying nothing (the option $(0, 0, 0)$) is always considered by the individual, represented by the fact that $(0, 0, 0) \in Q(p, q, b)$ for any $(p, q, b) \in \mathbb{R}^2_+ \times \{0, 1\}$.[5] This is motivated by the fact that 'not buying' is a special option for the decision maker, which could always be considered. Moreover, since often purchases can be returned, we could imagine that if our decision maker were to purchase an option that she wouldn't choose against $(0, 0, 0)$, she could always return it – thus such sales should not be considered. It should be noted that if we were to drop this assumption, our results would be

[4]Since this model coincides with rational choice when $r(S) = \Diamond$ for all S, then we can also assume that *all* types are modeled as in Section 3, but that for types H and L we also have $r(S) = \Diamond$ for all S.

[5]Other choices for the correspondence Q that yield narrower attraction regions (and hence weaker attraction effects) may also be considered here. While our definition of Q yields a particularly tractable model, the subsequent analysis would remain qualitatively unaltered with such alternative specifications.

substantially different – as the monopolist would exploit the possibility of selling objects with a negative utility – but the main qualitative findings of our analysis would remain unaltered.

Finally, we discuss the choice of $\mathbf{r}_i$. Recall that in the reference dependent choice model, for a given set S, we can have $\mathbf{r}_i(S) \in S$ only if there exists $(p, q, b) \in S \cap Q(\mathbf{r}_i(S))$ with $U_{A_i}(p, q) > U_{A_i}(\mathbf{r}_i(S))$: a reference can come into play only if it attracts the attention of the individual to some alternative that is strictly better than the reference itself. We then define $\mathbf{r}(S)$ as to be an arbitrary member of the dominated alternatives in S, and set it to $\diamondsuit$ (and hence posit reference-free behavior) if there is no such alternative. For concreteness, for any finite S, we shall take $\mathbf{r}(S)$ to be a bundle that maximizes U_{A_i} among all the *dominated* alternatives in S (that is, all alternatives $z \in S$ such that $U_{A_i}(x) > U_{A_i}(z)$ for some $x \in Q(z) \cap S$).[6]

This completes the specification of $\langle U_{A_i}, \mathbf{r}_i, Q\rangle$, that is, the choice behavior of a consumer who suffers from the attraction effect. It is easy to check that $\langle U_{A_i}, \mathbf{r}_i, Q\rangle$ is indeed a reference-dependent choice model in the sense of Definition 3. We denote the choice correspondence that is represented by this model as c_{A_i}.

4.2 Stating the Problem

Now that we have specified the choice correspondence of the agents who suffer from the attraction effect, we can study the menu of bundles offered by the monopolist in the presence of such agents. We assume that for each $i = L, H$, a fraction $\alpha_i \in [0, 1]$ of the i type agents suffers from the attraction effect. We first observe that it is never going to be the case that the monopolist will want to offer a menu with more than six different goods. In fact, we can write the menu offered by the monopolist as $\{(p_L, q_L, b_L)$, $(p_{A_L}, q_{A_L}, b_{A_L})$, $(p_{R_L}, q_{R_L}, b_{R_L})$, (p_H, q_H, b_H), $(p_{A_H}, q_{A_H}, b_{A_H})$, $(p_{R_H}, q_{R_H}, b_{R_H})\}$, where (p_L, q_L, b_L) , (p_H, q_H, b_L) are the bundles offered to the standard L and H types, $(p_{A_L}, q_{A_L}, b_{A_L})$, $(p_{A_H}, q_{A_H}, b_{A_H})$ are the bundles offered to the bounded rational versions of those types, and $(p_{R_L}, q_{R_L}, b_{R_L})$, $(p_{R_H}, q_{R_H}, b_{R_H})$ are the decoy goods used to attract the bounded rational agents of each type. Indeed there is no need for the monopolist to produce any additional product. We also understand that the bundle $(0, 0, 0)$, representing the act of not buying, is always available.

Before writing the monopolist's problem, we need to specify the cost of producing the decoy bundles $(p_{R_L}, q_{R_L}, b_{R_L})$ and $(p_{R_H}, q_{R_H}, b_{R_H})$. In fact, the main characteristic of these bundles is that they need to be observable by the agents when they make their choice and, therefore, they need to be produced. At the same time, however, they are *never sold* – to act as a decoy, they must be dominated by some other option. Indeed, how costly they are depends on the specific market, and we will assume that if the monopolist wants a product to be seen by a fraction α_i of the i type agents she has to incur a cost $\gamma\alpha_i q^2_{R_i}$. Indeed, γ will be equal to 1 if the monopolist needs to produce one decoy bundle for each consumer she wants to attract, while it will be equal to 0 in the cases in which the bundle in fact need not be actually produced.[7] Let $S := \{(p_L, q_L, b_L)$, $(p_{A_L}, q_{A_L}, b_{A_L})$, $(p_{R_L}, q_{R_L}, b_{R_L})$, (p_H, q_H, b_H),

[6]If there is more than one of such alternatives with the same utility pick the one with the smaller price. If they only differ in the third attribute pick the one with $b = 0$. If $|S| = \infty$, let $r(S) = \diamond$.

[7]We would have $\gamma = 1$ for a product sold in many locations, when the producer needs to offer a decoy bundle for all of them. By contrast, we would have $\gamma = 0$ for a good made to be ordered online: the decoy bundle, which is never sold, in this case is never actually produced.

$(p_{A_H}, q_{A_H}, b_{A_H})$, $(p_{R_H}, q_{R_H}, b_{R_H})$, $(0,0,0)\}$. The monopolist's problem in this case will be:

$$\Pi^* = \max \sum_{i=L,H} \left[(1-\alpha_i)\left(p_i - q_i^2\right) + \alpha_i (p_{A_i} - q_{A_i}^2) - \gamma \alpha_i q_{R_i}^2 \right]$$

$$\begin{aligned} \text{such that} \quad & U_L(p_L, q_L) \geq 0, \\ & U_H(p_H, q_H) \geq \max\{U_H(p_L, q_L), U_H(p_{A_L}, q_{A_L}), U_H(p_{A_H}, q_{A_H})\}, \\ & U_L(p_L, q_L) \geq \max\{U_L(p_H, q_H), U_L(p_{A_L}, q_{A_L}), U_L(p_{A_H}, q_{A_H})\}, \\ & (p_{A_L}, q_{A_L}, b_{A_L}) \in c_{A_L}(S), \\ & (p_{A_H}, q_{A_H}, b_{A_H}) \in c_{A_H}(S), \\ & p_{A_H}, q_{A_H}, p_{A_L}, q_{A_L}, p_H, q_H, p_L, q_L, p_{R_H}, q_{R_H}, p_{R_L}, q_{R_L} \geq 0. \end{aligned} \tag{4}$$

where we understand that the reference bundles $(p_{R_i}, q_{R_i}, b_{R_i})$ will be set as $(0,0,0)$ if $\mathbf{r}_{A_i}(S) = \diamond$. The first three constraints are in fact the standard participation and incentive compatibility constraints for the two groups of standard agents, H and L.[8] Indeed, the interesting constraints here are the fourth and the fifth ones.

At first, this problem might appear difficult to analize. We will show, however, that it can in fact be reduced to a much simpler problem that we will be able to handle with the usual methods. The first step in this direction is to notice that the monopolist will not have any motivation to exploit the attraction effect for the low types: this result is contatined in the following proposition.

Proposition 1. The problem above has a solution, and any solution for the problem above satisfies $(p_L, q_L) = (p_{A_L}, q_{A_L})$.

The intuition for this proposition is extremely simple and provides many insights to what will follow. First, consider what are the advantages gained by a monopolist who does choose to exploit the attraction effect for some agents: it allows her to relax the incentive compatibility constraint for a portion of the agents. In fact, this is the only advantage gained by the monopolist from the use of the attraction effect: by forcing the agents to focus only on some particular set of options, she can reduce the incentive problems. At the same time, it does not allow the monopolist to overcome any participation constraint, since the bundle to which the agents are attracted to must still offer a non-negative utility – the option of not-buying is always available. Now, would a monopolist want to exploit the attraction effect for the low type agents? On the one side, this has some costs: she needs to produce a (potentially) costly reference bundle, which itself might create new incentive problems for the other types. On the other side, the advantage is that she can then relax the incentive compatibility constraint for this fraction of the lower types. But as it is the case in the MR solution, and as we shall see it is still the case here, in the optimum the incentive compatibility constraints of the low types are *not binding* and, therefore, the monopolist has nothing to gain from relaxing them. Hence, she will never choose to exploit the attraction effect for the low types.

One should note that this result has some rather concrete marketing implications. In fact, this suggests that we should not see the exploitation of the attraction effect for the low types,

[8] As usual, we don't have to write the participation constraint of the H type, since it is implied by the participation constraint of L type plus the incentive compatibility constraint of the H type.

or, more in general, for the fraction of the agents whose incentive compatibility constraints are not binding in the solution.

Proposition 1 shows that we can concentrate our attention on the menus that do not use a decoy bundle for the low type bounded rational agents. That is, we can concentrate on the situations where the monopolist solves the following problem:

$$\begin{aligned}
\Pi^* = \max \quad & p_L - q_L^2 + \left[(1-\alpha_H)\left(p_H - q_H^2\right) + \alpha_H (p_{A_H} - q_{A_H}^2) - \gamma \alpha_H q_{R_H}^2\right] \\
\text{such that} \quad & U_L(p_L, q_L) \geq 0, \\
& U_H(p_H, q_H) \geq \max\{U_H(p_L, q_L), U_H(p_{A_H}, q_{A_H})\}, \\
& U_L(p_L, q_L) \geq \max\{U_L(p_H, q_H), U_L(p_{A_H}, q_{A_H})\}, \\
& (p_{A_H}, q_{A_H}, b_{A_H}) \in c_{A_H}(S), \\
& p_{A_H}, q_{A_H}, p_H, q_H, p_L, q_L, p_{R_H}, q_{R_H} \geq 0.
\end{aligned} \tag{5}$$

where now $S := \{(p_L, q_L, b_L), (p_H, q_H, b_H), (p_{A_H}, q_{A_H}, b_{A_H}), (p_{R_H}, q_{R_H}, b_{R_H}), (0,0)\}$. Moreover, it turns out that we can simplify the problem even further: the following relaxed version of the problem has the same solutions of the problem above:

$$\begin{aligned}
\Pi^* = \max \quad & p_L - q_L^2 + \left[(1-\alpha_H)\left(p_H - q_H^2\right) + \alpha_H (p_{A_H} - q_{A_H}^2) - \gamma \alpha_H q_{R_H}^2\right] \\
\text{such that} \quad & U_L(p_L, q_L) \geq 0, \\
& U_H(p_H, q_H) \geq U_H(p_L, q_L), \\
& (p_{A_H}, q_{A_H}, b_{A_H}) \in c_{A_H}(S).
\end{aligned} \tag{6}$$

4.3 Solution

4.3.1 General Solution

The detailed solution of the problem at hand, divided in four cases, is discussed at length in the appendix. The main results, however, appear in the following two propositions.

Proposition 2. For any $\gamma \in [0, 1]$, there exists a $\underline{\alpha} \in [0, 1)$, such that it is strictly more profitable for the monopolist to use a decoy good than to use the *MR* solution if and only if $\alpha_H > \underline{\alpha}$.

Proposition 3. For any $\alpha_H \in (0, 1)$, there exists a $\bar{\gamma} > 0$, such that using a decoy good is strictly more profitable than the *MR* solution if and only if $\gamma < \bar{\gamma}$.

Proposition 2 shows that, for any admissible values of θ_L, θ_H, and for any possible cost γ, there exists a threshold $\underline{\alpha} \in [0, 1)$ such that the monopolist will want to exploit the attraction effect if, and only if, the proportion of the 'high' types who are subject to the attraction effect is above that threshold, i.e. if $\alpha_H > \underline{\alpha}$. In turn, since $\underline{\alpha} < 1$, this means that, no matter what the cost γ is, there always exists a proportion α such that the monopolist will find it *strictly* profitable to exploit the attraction effect. In particular, if all high types are subject to the attraction effect ($\alpha_H = 1$), then the monopolist will *always* exploit it.

The intuition of this result is simple. Recall that the attraction effect could be exploited by the monopolist to circumvent the incentive-compatibility constraints of the customers who are subject to it. The cost of doing this, however, is to pay the cost in producing the decoy good, and to add an *additional* constraint for the customers who are not subject to it. Indeed if this latter group is small, the monopolist will accept this to extract more rent from those who are subject to the attraction effect. But if this group is large, then the situation is reversed, and the monopolist will not want to do so.

Proposition 3 discusses a similar argument for the cost γ: no matter what α_H is, there always exists a threshold $\bar{\gamma}$, strictly positive, such that the monopolist will want to exploit the attraction effect if, and only if the cost is below the threshold, i.e. $\gamma < \bar{\gamma}$. In turn, since $\bar{\gamma}$ is strictly positive, this means that there always exists a cost low enough that the monopolist will want to exploit the attraction effect, no matter what α_H is. In particular, if $\gamma = 0$, then the monopolist will *always* want to the exploit the attraction effect, even if α_H is arbitrarily small.

The two propositions above show that at least when the cost of using a decoy good is not too large or the number of bounded rational agents is high enough, the monopolist will make use of such a strategy. We now show that except for a small range of the parameters θ_L and θ_H the use of a decoy good will be profitable for the monopolist for all values of $\alpha \in (0, 1)$ and $\gamma \in [0, 1]$.[9]

4.3.2 A special case: $\alpha_H = 1$ and $\gamma = 0$

An important special case of our analysis is the one in which $\alpha_H = 1$ and $\gamma = 0$: this is the case in which every high type is subject to the attraction effect, and in which the monopolist incurs no cost in producing a decoy bundle – as it is the case for online sales. In this situation, as we can see from Case 1's solution in the appendix, the monopolist can induce both the low type and the A_H type to consume the *efficient* amount. This means that, as opposed to standard screening models, the economy reaches the first best solution, with the monopolist extracting the entire surplus.

The reason is, in the MR model the monopolist could not reach the profits she would obtain were she able to segment the market because of the incentive compatibility constraints of the H type. But here, by exploiting only its attraction effect, the monopolist is able to overcome this incentive compatibility constraint, and therefore extract all the revenue from the consumers, reaching her first best solution. To do this, she only has to offer a decoy bundle $(p_{R_H}, q_{R_H}, b_{R_H})$ to prevent the incentive compatibility constraint of the H type to matter. And since $\gamma = 0$, this decoy bundle can be produced at no cost, and, thus, the efficient solution can be attained. In particular, she could produce the following bundles, depicted in Figure 4:

$$q^*_{A_H} := \frac{\theta_H}{2},\ q^*_L := \frac{\theta_L}{2},\ q_{R_H} = \frac{\theta_H + \theta_L}{2}$$

with prices such that the participation constraints of both types H and L are binding, and

[9] It turns out that except for a small range of the parameters θ_L and θ_H the use of a decoy good will be profitable for the monopolist for all values of $\alpha \in (0, 1)$ and $\gamma \in [0, 1]$.
Proposition 4. There exists $\frac{1}{2} < \underline{a} < \overline{a} < 1$ such that if $\theta_L \leq \underline{a}\theta_H$ or $\theta_L \geq \overline{a}\theta_H$, the monopolist will use a decoy good for any $\alpha \in (0, 1)$ and any $\gamma \in [0, 1]$.

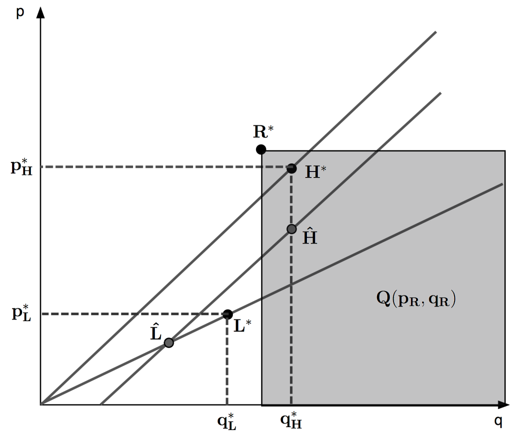

Figure 4. Solution of the model when $\alpha_H = 1$ and $\gamma = 0$.

any $p_{R_H} > p_{A_H}$. Hence, the following proposition.

Proposition 5. If all high type agents are subject to the attraction effect ($\alpha_H = 1$) and the monopolist bears no cost in producing a bundle that is offered but never sold ($\gamma = 0$), then the monopolist is able to perfectly segment the market, the solution is efficient, and the monopolist extracts all the surplus.

4.4 Welfare Analysis

We now analyze the welfare implications of our model. In particular, we analyze how the welfare of each player in the economy changes as α_H changes. For simplicity, we focus only on the case $\gamma = 0$. In this case, as we have seen, the monopolist will always choose to exploit the attraction effect and her profit will be higher than in the MR case. In particular, when $\gamma = 0$, it can be checked that the expression for the profits of the monopolist comes from case 1 in the appendix and is given by

$$\frac{\left(\theta_L - (1-\alpha_H)\left(\theta_H - \theta_L\right)\right)^2}{4} + \frac{\theta_H^2}{4},$$

which is increasing and convex in α_H. At the same time, this increase in profits happens at the expenses of some of the consumers, who are now "boundedly rational" since they are subject to the attraction effect. One should, therefore, expect the consumers to be worse off the higher α_H is. But in fact, this is not always the case.

We now turn to analyze the consumer's welfare. First of all, notice that the consumers of the low type always get the same welfare, equal to zero, no matter what α_H is, since their participation constraint is always binding for any parameter value. The key point of the analysis is what happens to the consumers of the high type. Indeed, if $\alpha_H > 0$, they will be divided into two groups, A_H and H, exactly in an α_H and $(1-\alpha_H)$ proportions. The welfare of type H when $\gamma = 0$ is

$$(\theta_H - \theta_L)(-\frac{1}{2}(1-\alpha_H)(\theta_H - \theta_L) + \frac{\theta_L}{2}),$$

which is increasing and linear in α_H. This means that each agent of the high type who does *not* suffer from the attraction effect is actually *better off* the more people in her group do suffer from the attraction effect. The reason is rather simple: the less people of type H are in the market, the less the monopolist will want to distort her offerings to the other types in order to extract more revenue from them, and therefore the better off they are. Finally, we are left with customers of type A_H. It is immediate to see that any such agent is indifferent to how many other agents suffer from the attraction effect: in fact, her participation constraint is always binding, and therefore her welfare is always zero.

The discussion above seems to suggest that, as α_H increases, everybody is weakly better off: the monopolists and type H are strictly better off, while type A_H and type L are indifferent. But clearly this is not the case: an increase in α_H means that more and more agents do suffer from the attraction effect, that is, some agents go from being type H (where they had a positive payoff) to being type A_H (where they have a zero payoff). And if we wish to be able to analyze the total welfare, we would need a way to compare their welfare as they change their status, but of course this is not trivial, as with their type they also change their preferences. In what follows, we will take the simplest approach: for those subjects who go from being type H to being type A_H we consider their change in welfare as the difference in welfare of the two types computed using their respective preferences. In our case this is particularly simple: since type A_H receives a welfare zero, their change in welfare is the welfare they had as H, which they loose becoming A_H.[10] This means that all such agents are actually *worse off* as α_H increases. In fact, they are the only ones loosing from the change.

We can now compute both the total welfare, and the aggregate consumer welfare. Let us start from the latter. Since agents of type L always receive zero, what changes is the aggregate welfare of the agents of the high type (H and A_H).[11] It turns out that, if θ_H and θ_L are different enough, and if α_H is small enough, their welfare is actually increasing with α_H.[12] This means that, as long as α_H is small, a consumer is actually *better off* if her chances of being boundedly rational *increase*. This happens because the welfare increase of types H more than compensates the welfare loss of some customers going from H to A_H. Put differently, the move towards efficiency, which takes place as α_H increases, is such that, on average, consumers are also better off. In turn, this means that in this parametric range, an increase in the bounded rationality of the agents (increase in α_H) is actually leading to an improvement *in expected terms* for each member of the economy, since both the monopolist

[10]Indeed other notions are possible, but here we are taking the most conservative one in terms of total welfare, i.e. we are taking the approach for which the welfare increases as little as possible as α_H increases – making our results of a welfare increase robust to other specifications. This approach, moreover, is justified by the fact that type A_H has indeed the possibility to buy the bundle that type H buys, but she doesn't do so exactly because she is subject to the attraction effect. If, however, other bundles were not present, and she were offered only those two, she would instead buy the bundle that H buys, since there are no reference effects with only two options. Again this suggests that the utility lost from buying H is exactly the welfare lost passing from being H to being A_H.

[11]If the model is interpreted as there being only *one* agent, whose type is unknown to the monopolist, then what we now describe is the *ex-ante* welfare of such customer, *before* she finds out her type.

[12]More precisely, the total consumer welfare is increasing in α as long as $\theta_L < \frac{2}{3}\theta_H$ and $\alpha < \frac{2\theta_H - 3\theta_L}{2\theta_H - 2\theta_L}$.

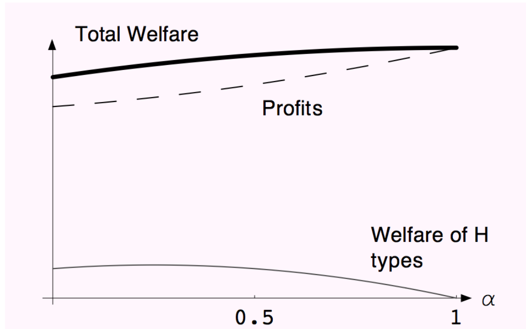

Figure 5. Welfare changes with α_H, computed with $\gamma = 0$, $\theta_L = 0.6$, $\theta_H = 1$ and $\alpha_H \in [0, 1]$.

and the high type agents are better off, while the low type agents are indifferent.

Finally, we consider the change in total welfare. It turns out that the total welfare is a strictly increasing and concave function of α_H. Part of this result is intuitive: since the higher the proportion of agents who suffer from the attraction effect, the closer the market gets to the efficient allocation (which, again, is met when $\alpha_H = 1$). Graphically, an example of the evolution of the welfare, both aggregate and of the specific players, is depicted in Figure 5 (where we have $\gamma = 0$, $\theta_L = 0.6$ and $\theta_H = 1$, while the MR solution is the one for $\alpha_H = 0$).

The discussion above can be summarized in the following proposition:

Proposition 6. *If* $\gamma = 0$:

1. *The profit of the monopolist is a striclty increasing and convex function of* α_H.
2. *The welfare of type L is unchanged with* α_H *and is always equal to 0.*
3. *The welfare of type H who remain type H is increasing and linear in* α_H.
4. *The welfare of type* A_H *who remain type* A_H *is constant and equal to zero.*
5. *If* $\theta_L < \frac{2}{3}\theta_H$*, the expected welfare of the aggregate of types H and* A_H*, as well as of the consumers as a whole is increasing in* α_H *as long as* $\alpha_H < \frac{2\theta_H - 3\theta_L}{2\theta_H - 2\theta_L}$*, decreasing for bigger values, and it reaches a strictly lowest value at* $\alpha_H = 1$*. If* $\theta_L \geq \frac{2}{3}\theta_H$*, it is a decreasing function of* α_H.
6. *Total welfare is a strictly increasing and concave function of* α_H.

Appendix: Product Differentiation with Attraction Effect

Solution of the Simplified Problem

Consider problem (6) in the main text. We start from a simple observation:

Observation 1. If a menu satisfies the first three constraints of the problem above and it gives a payoff at least as great as the *MR* solution, then it must necessarily satisfy $p_{A_H} - q_{A_H}^2 - \gamma q_{R_H}^2 \geq p_H - q_H^2$. If such menu gives a payoff strictly greater than the *MR* solution, then the inequality above must be strict.

Proof of Observation 1. Suppose that a given menu satisfies all the constraints of the problem above and $p_{A_H} - q_{A_H}^2 - \gamma q_{R_H}^2 \overset{(\leq)}{<} p_H - q_H^2$. But this implies that $(p_H, q_H), (p_L, q_L)$ also satisfy the constraints of the *MR* problem, which implies that

$$\begin{aligned}\Pi &= \left(p_L - q_L^2\right) + \alpha_H\left(p_{A_H} - q_{A_H}^2\right) + (1-\alpha_H)\left(p_H - q_H^2\right) - \gamma\alpha_H q_{R_H}^2 \\ &\overset{(\leq)}{<} \left(p_L - q_L^2\right) + \left(p_H - q_H^2\right) \\ &\leq \Pi^{MR}\end{aligned}$$

as we sought ‖

We now turn to discuss the solutions of the problem below. It can be divided in four cases:

Case 0. Suppose we add the constraint $U_H(p_{A_H}, q_{A_H}) \geq U_H(p_L, q_L)$ to the problem. The problem becomes

$$\Pi^* = \max \quad p_L - q_L^2 + \left[(1-\alpha_H)\left(p_H - q_H^2\right) + \alpha_H(p_{A_H} - q_{A_H}^2) - \gamma\alpha_H q_{R_H}^2\right]$$

$$\text{such that} \quad \begin{aligned} &U_L(p_L, q_L) \geq 0, \\ &U_H(p_H, q_H) \geq U_H(p_L, q_L), \\ &U_H(p_{A_H}, q_{A_H}) \geq U_H(p_L, q_L), \\ &(p_{A_H}, q_{A_H}, b_{A_H}) \in c_{A_H}(S). \end{aligned}$$

We work with the following relaxed version of the problem above:

$$\Pi^* = \max \quad p_L - q_L^2 + \left[(1-\alpha_H)\left(p_H - q_H^2\right) + \alpha_H(p_{A_H} - q_{A_H}^2) - \gamma\alpha_H q_{R_H}^2\right]$$

$$\text{such that} \quad \begin{aligned} &U_L(p_L, q_L) \geq 0, \\ &U_H(p_H, q_H) \geq U_H(p_L, q_L), \\ &U_H(p_{A_H}, q_{A_H}) \geq U_H(p_L, q_L). \end{aligned}$$

It can be checked that all solutions of the problem above agree with the MR solution, in the sense that $q_{R_H} = 0$ and (p_L, q_L), $(p_H, q_H) = (p_{A_H}, q_{A_H})$ are exactly the values found in that solution.

We have just shown that it is never going to be the case that the monopolist will exploit the attraction effect in order to make the individuals choose a good $(p_{A_H}, q_{A_H}, b_{A_H})$ such that $U_H(p_{A_H}, q_{A_H}) \geq U_H(p_L, q_L)$. This shows that the remaining interesting cases are the ones in which $U_H(p_{A_H}, q_{A_H}) < U_H(p_L, q_L) \leq U_H(p_H, q_H)$. This can happen only if $Q(\mathbf{r}_{A_H}(S)) \cap S = \{(p_{A_H}, q_{A_H}, b_{A_H}), (p_{R_H}, q_{R_H}, b_{R_H}), (0, 0)\}$. This allows us to write the problem as

$$\Pi^* = \max \quad p_L - q_L^2 + \left[(1-\alpha_H)\left(p_H - q_H^2\right) + \alpha_H(p_{A_H} - q_{A_H}^2) - \gamma\alpha_H q_{R_H}^2\right]$$

$$\text{such that} \quad \begin{aligned} &U_L(p_L, q_L) \geq 0, \\ &U_H(p_H, q_H) \geq U_H(p_L, q_L), \\ &p_{A_H} \leq p_{R_H},\ q_{A_H} \geq q_{R_H} \text{ and } U_H(p_{A_H}, q_{A_H}) \geq 0, \\ &p_H \geq p_{R_H} \text{ or } q_H \leq q_{R_H}, \\ &p_L \geq p_{R_H} \text{ or } q_L \leq q_{R_H}. \end{aligned}$$

Notice that in writting the problem above we are using the fact that the monopolist can use the third attribute to decide if an alternative that weakly dominates the reference with respect to the first two attributes belongs or not to $Q(\mathbf{r}(S))$. We can divide the problem above in three cases.

Case 1. Suppose that we impose that the constraints $q_H \leq q_{R_H}$ and $q_L \leq q_{R_H}$ are satisfied. We are left with the following problem:

$$\begin{aligned} \Pi^* = \max \quad & p_L - q_L^2 + \left[(1-\alpha_H)(p_H - q_H^2) + \alpha_H(p_{A_H} - q_{A_H}^2) - \gamma\alpha_H q_{R_H}^2\right] \\ \text{such that} \quad & U_L(p_L, q_L) \geq 0, \\ & U_H(p_H, q_H) \geq U_H(p_L, q_L), \\ & p_{A_H} \leq p_{R_H},\ q_{A_H} \geq q_{R_H} \text{ and } U_H(p_{A_H}, q_{A_H}) \geq 0, \\ & q_H \leq q_{R_H}, \\ & q_L \leq q_{R_H}. \end{aligned}$$

In what follows we work with the following relaxed version of the problem above

$$\begin{aligned} \Pi^* = \max \quad & p_L - q_L^2 + \left[(1-\alpha_H)(p_H - q_H^2) + \alpha_H(p_{A_H} - q_{A_H}^2) - \gamma\alpha_H q_{R_H}^2\right] \\ \text{such that} \quad & p_L - \theta_L q_L \geq 0, \\ & p_H - \theta_H q_H \geq p_L - \theta_H q_L, \\ & p_{A_H} - \theta_H q_{A_H} \geq 0, \\ & q_H \leq q_{R_H}, \\ & q_L \leq q_{R_H}. \end{aligned}$$

It is easy to see that the first three constraints must be binding. The problem reduces to

$$\begin{aligned} \Pi^* = \max \quad & \theta_L q_L - q_L^2 + \left[(1-\alpha_H)(\theta_H q_H - (\theta_H - \theta_L)q_L - q_H^2) + \alpha_H(\theta_H q_{A_H} - q_{A_H}^2) - \gamma\alpha_H q_{R_H}^2\right] \\ \text{such that} \quad & q_H \leq q_{R_H}, \\ & q_L \leq q_{R_H}. \end{aligned}$$

It is not hard to see that the first constraint must be binding.[13] We can write the problem above as

$$\begin{aligned} \Pi^* = \max \quad & \theta_L q_L - q_L^2 + \left[(1-\alpha_H)(\theta_H q_H - (\theta_H - \theta_L)q_L - q_H^2) + \alpha_H(\theta_H q_{A_H} - q_{A_H}^2) - \gamma\alpha_H q_H^2\right] \\ \text{such that} \quad & q_L \leq q_H. \end{aligned}$$

The solution for this problem is

$$q_L = \frac{\theta_L - (1-\alpha_H)(\theta_H - \theta_L)}{2},$$

$$q_H = \frac{1-\alpha_H}{1-\alpha_H+\gamma\alpha_H}\frac{\theta_H}{2},$$

$$q_{A_H} = \frac{\theta_H}{2},$$

if

$$\gamma \leq \frac{\left[1-\alpha_H + (1-\alpha_H)^2\right](\theta_H - \theta_L)}{\alpha_H\left[\theta_L - (1-\alpha_H)(\theta_H - \theta_L)\right]},$$

and

$$q_L = q_H = \frac{2-\alpha_H}{2-\alpha_H+\gamma\alpha_H}\frac{\theta_L}{2},$$

$$q_A = \frac{\theta_H}{2},$$

otherwise. The monopolist's profit is

$$\Pi = \frac{(\theta_L - (1-\alpha_H)(\theta_H - \theta_L))^2}{4} + \frac{1-\alpha_H+\gamma\alpha_H^2}{1-\alpha_H+\gamma\alpha_H}\frac{\theta_H^2}{4},$$

[13] If this was not true we would have $q_H = \frac{\theta_H}{2} < q_{R_H}$. Independently of the value of q_L, a deviation to $\hat{q}_H = q_H$, $\hat{q}_L = \frac{\theta_L - (1-\alpha_H)(\theta_H - \theta_L)}{2}$ and $\hat{q}_{R_H} = q_H$ would yield the monopolist a higher profit.

in the first case and

$$\Pi = \alpha_H \frac{\theta_H^2}{4} + \frac{(2-\alpha_H)^2}{2-\alpha_H+\gamma\alpha_H}\frac{\theta_L^2}{4},$$

in the second case. One can check that both solutions satisfy all the constraints that were ignored in order to get to the final simplified problem.

Case 2. Now suppose that we impose that the constraints $p_H \geq p_{R_H}$ and $q_L \leq q_{R_H}$ are satisfied. We first note that the precise value of p_{R_H} is not really important for the problem, all that matters is that it lies between p_{A_H} and p_H, so we can collapse the constraints $p_{A_H} \leq p_{R_H}$ and $p_{R_H} \leq p_H$ into $p_{A_H} \leq p_H$. Also, it is clear that the solution will necessarily have $q_{R_H} = q_L$. We are left with the following problem:

$$\begin{aligned}
\Pi^* = \max \quad & p_L - q_L^2 + \left[(1-\alpha_H)(p_H - q_H^2) + \alpha_H(p_{A_H} - q_{A_H}^2) - \gamma\alpha_H q_L^2\right] \\
\text{such that} \quad & U_L(p_L, q_L) \geq 0, \\
& U_H(p_H, q_H) \geq U_H(p_L, q_L), \\
& U_H(p_{A_H}, q_{A_H}) \geq 0, \\
& p_H \geq p_{A_H}, \\
& q_{A_H} \geq q_L.
\end{aligned}$$

We look to the following relaxed version of the problem above:

$$\begin{aligned}
\Pi^* = \max \quad & p_L - q_L^2 + \left[(1-\alpha_H)(p_H - q_H^2) + \alpha_H(p_{A_H} - q_{A_H}^2) - \gamma\alpha_H q_L^2\right] \\
\text{such that} \quad & \theta_L q_L - p_L \geq 0, \\
& \theta_H q_H - p_H \geq \theta_H q_L - p_L, \\
& \theta_H q_{A_H} - p_{A_H} \geq 0, \\
& p_H \geq p_{A_H}.
\end{aligned}$$

It is easy to see that all four constraints must be binding, which gives us the following simplified problem:

$$\begin{aligned}
\Pi^* = \max \quad & \theta_L q_L - q_L^2 + \left[(1-\alpha_H)(\theta_H q_H - (\theta_H - \theta_L) q_L - q_H^2) + \alpha_H(\theta_H q_{A_H} - q_{A_H}^2) - \gamma\alpha_H q_L^2\right] \\
\text{such that} \quad & \theta_H q_{A_H} = \theta_H q_H - (\theta_H - \theta_L) q_L.
\end{aligned}$$

The solution for such problem is

$$q_L = \left(\frac{\theta_L - (1-\alpha_H)(\theta_H - \theta_L)}{2}\right) \Big/ \left(1 + \gamma\alpha_H + \frac{\alpha_H(1-\alpha_H)(\theta_H-\theta_L)^2}{\theta_H^2}\right),$$

$$q_H = \frac{\theta_H}{2} + \frac{\alpha_H(\theta_H - \theta_L)}{\theta_H} q_L,$$

$$q_{A_H} = \frac{\theta_H}{2} - \frac{(1-\alpha_H)(\theta_H - \theta_L)}{\theta_H} q_L,$$

and the monopolist's profit is

$$\Pi^* = \frac{\theta_H^2}{4} + ((2-\alpha_H)\theta_L - (1-\alpha_H)\theta_H) q_L - \left(1 + \gamma\alpha_H + \frac{\alpha_H(1-\alpha_H)(\theta_H-\theta_L)^2}{\theta_H^2}\right) q_L^2.$$

One can check that all constraints that were ignored in order to get to the simplified problem above are satisfied by this solution.

Case 3. Suppose now that we impose that the constraint $p_{R_H} \le p_L$ is satisfied. We are left with the following problem:

$$\Pi^* = \max \quad p_L - q_L^2 + \left[(1-\alpha_H)\left(p_H - q_H^2\right) + \alpha_H (p_{A_H} - q_{A_H}^2) - \gamma \alpha_H q_{R_H}^2 \right]$$

$$\begin{aligned} \text{such that} \quad & U_L(p_L, q_L) \ge 0, \\ & U_H(p_H, q_H) \ge U_H(p_L, q_L), \\ & p_{A_H} \le p_{R_H},\ q_{A_H} \ge q_{R_H} \text{ and } U_H(p_{A_H}, q_{A_H}) \ge 0, \\ & p_H \ge p_{R_H} \text{ or } q_H \le q_{R_H}, \\ & p_L \ge p_{R_H}. \end{aligned}$$

We work with the following relaxed version of the problem above:

$$\Pi^* = \max \quad p_L - q_L^2 + \left[(1-\alpha_H)\left(p_H - q_H^2\right) + \alpha_H (p_{A_H} - q_{A_H}^2) - \gamma \alpha_H q_{R_H}^2 \right]$$

$$\begin{aligned} \text{such that} \quad & \theta_L q_L - p_L \ge 0, \\ & \theta_H q_H - p_H \ge \theta_H q_L - p_L, \\ & \theta_H q_{A_H} - p_{A_H} \ge 0, \\ & p_{A_H} \le p_L, \\ & q_{A_H} \ge q_{R_H}. \end{aligned}$$

Clearly, the solution for the problem above will have $q_{R_H} = 0$, and given that $q_{A_H} < 0$ will never be optimal, the constraint $q_{A_H} \ge q_{R_H}$ is automatically satisfied. With this result at hand it is clear that the first three constraints will be binding. In fact one can check that the fourth constraint will also be binding, which gives us the following problem:

$$\Pi^* = \max \quad \theta_L q_L - q_L^2 + \left[(1-\alpha_H)\left(\theta_H q_H - (\theta_H - \theta_L)\, q_L - q_H^2\right) + \alpha_H (\theta_H q_{A_H} - q_{A_H}^2) - \gamma \alpha_H q_{R_H}^2 \right]$$

$$\text{such that} \quad \theta_H q_{A_H} = \theta_L q_L.$$

The solution for such problem is

$$\begin{aligned} q_L &= \frac{\theta_H}{2} \frac{\theta_H \left(2\theta_L - (1-\alpha_H)\,\theta_H\right)}{\theta_H^2 + \alpha_H \theta_L^2}, \\ q_{A_H} &= \frac{\theta_H}{2} \frac{\theta_L \left(2\theta_L - (1-\alpha_H)\,\theta_H\right)}{\theta_H^2 + \alpha_H \theta_L^2}, \\ q_H &= \frac{\theta_H}{2}, \end{aligned}$$

and the monopolist's profit is

$$\Pi = \frac{\theta_H^2}{4} \frac{\left(2\theta_L - (1-\alpha_H)\,\theta_H\right)^2}{\theta_H^2 + \alpha_H \theta_L^2} + \frac{(1-\alpha_H)\,\theta_H^2}{4}.$$

One can check that all the constraints that were ignored in order to get to the simplified problem above are satisfied by this solution.

Proof of Proposition 1.

Consider the following relaxed version of the problem in the main text

$$\Pi^* = \max \quad \sum_{i=L,H} \left[(1-\alpha_i)\left(p_i - q_i^2\right) + \alpha_i (p_{A_i} - q_{A_i}^2) - \gamma \alpha_i q_{R_i}^2 \right]$$

$$\begin{aligned} \text{such that} \quad & U_L(p_L, q_L) \ge 0, \\ & U_H(p_H, q_H) \ge \max\{U_H(p_L, q_L), U_H(p_{A_L}, q_{A_L})\}, \\ & U_L(p_{A_L}, q_{A_L}) \ge 0, \\ & (p_{A_H}, q_{A_H}, b_{A_H}) \in c_{A_H}(S). \end{aligned}$$

We note that $(p_{R_L}, q_{R_L}, b_{R_L})$ does not affect any of the constraints of the problem above, so clearly any solution for the problem above must satisfy $(p_{R_L}, q_{R_L}) = (0,0)$. Also, since (p_L, q_L) and (p_{A_L}, q_{A_L}) are subject to the same restrictions, we can use the concavity of the objective function together with the convexity of the restriction set to show that any solution to the problem above must satisfy $(p_L, q_L) = (p_{A_L}, q_{A_L})$. This allows us to concentrate on the following simplified version of the problem above:

$$\begin{aligned} \Pi^* = \max \quad & p_L - q_L^2 + \left[(1-\alpha_H)(p_H - q_H^2) + \alpha_H(p_{A_H} - q_{A_H}^2) - \gamma\alpha_H q_{R_H}^2\right] \\ \text{such that} \quad & U_L(p_L, q_L) \geq 0, \\ & U_H(p_H, q_H) \geq U_H(p_L, q_L), \\ & (p_{A_H}, q_{A_H}, b_{A_H}) \in c_{A_H}(S). \end{aligned}$$

We have already shown that the problem above always has a solution that satisfies all the restrictions of the original problem. ‖

Proof of Proposition 2.

By Proposition 1 and the solution for the simplified problem, we know that the solution of the original problem always agrees with the solution of the simplified problem.

Let's first show that for any $\theta_L, \theta_H > 0$ and $\gamma = 1$, if α_H is big enough, then the simplified problem is strictly more profitable than the *MR* case. Observe that in this case the payoff in Case 2 when $\alpha_H \to 1$ converges to

$$\Pi^{Case2} \to \frac{\theta_H^2}{4} + \frac{\theta_L^2}{8}.$$

Similarly, the payoff in Case 3 when $\alpha_H \to 1$ converges to

$$\Pi^{Case3} \to \frac{\theta_H^2}{4} \frac{4\theta_L^2}{\theta_H^2 + \theta_L^2}.$$

Suppose that the limit of the payoff in Case 2 is smaller than the payoff in the *MR* solution, that is,

$$\frac{\theta_H^2}{4} + \frac{\theta_L^2}{8} \leq \left(\theta_L - \frac{\theta_H}{2}\right)^2 + \frac{\theta_H^2}{4}.$$

Let $a = \frac{\theta_L}{\theta_H}$. The expression above is equivalent to

$$\frac{7a^2}{8} - a + \frac{1}{4} \geq 0.$$

On the other hand, the limit of the payoff in Case 3 is strictly larger than the payoff in the *MR* solution if, and only if,

$$a^4 - a^3 + \frac{a^2}{2} - a + \frac{1}{2} < 0.$$

Since one is a root of the polynomial above and $a \in \left(\frac{1}{2}, 1\right)$, the condition above will be true if and only if

$$a^3 + \frac{a}{2} - \frac{1}{2} > 0.$$

Finally, we note that the polynomial $\frac{7a^2}{8} - a + \frac{1}{4}$ has one root in $\left(0, \frac{1}{2}\right)$ and another root in $\left(\frac{3}{4}, 1\right)$. On the other hand the polynomial $a^3 + \frac{a}{2} - \frac{1}{2}$ has a single root which is located in $\left(\frac{1}{2}, \frac{3}{4}\right)$. Moreover $a^3 + \frac{a}{2} - \frac{1}{2} > 0$ if a is greater than this root. So whenever we have that Case 2 gives a profit smaller than the *MR* solution in the limit we must necessarily have that Case 3 gives a profit strictly greater than the *MR* solution when α_H approaches 1. We conclude that there always exists α_H such that $\Pi^{\text{Problem (2)}} > \Pi^{MR}$ and this is obviously also true when $\gamma < 1$. Define $\underline{\alpha} := \inf\left\{\alpha_H : \Pi^{\text{Problem (2)}} > \Pi^{MR}\right\}$. By what we have just proved $\underline{\alpha}$ is well-defined and it is easy to see that $\Pi^{\text{Problem (2)}} \leq \Pi^{MR}$ when $\alpha_H = \underline{\alpha}$. To complete the proof of the proposition we show that for any $\alpha_H > \underline{\alpha}$, $\Pi^{\text{Problem (2)}} > \Pi^{MR}$. For that pick any $\alpha^* > \underline{\alpha}$. By construction we know that there exists $\hat{\alpha} \in [\underline{\alpha}, \alpha^*)$ such that when $\alpha_H = \hat{\alpha}$, $\Pi^{\text{Problem (2)}} > \Pi^{MR}$. By observation 1 we know that this implies that $p_A - q_A^2 - \gamma q_R^2 > p_H - q_H^2$. But then, if the monopolist continues to offer this same bundle when $\alpha_H = \alpha^*$, all the constraints will still be satisfied and she is going to make a higher profit than before. Consequently, the solution when $\alpha_H = \alpha^*$ gives a strictly higher profit than the *MR* solution. This completes the proof of the proposition. ‖

Proof of Proposition 3.

For any $\alpha_H \in (0,1)$, Case 1 gives a payoff strictly higher than the *MR* solution when $\gamma = 0$. So define $\bar{\gamma} := \sup\left\{\gamma : \Pi^{\text{Problem (2)}} > \Pi^{MR}\right\}$. By what we have just discussed $\bar{\gamma} > 0$ and, moreover, it is easy to see that when $\gamma = \bar{\gamma}$ we must necessarily have $\Pi^{\text{Problem (2)}} \leq \Pi^{MR}$. It only remains to be shown that for any $\gamma < \bar{\gamma}$ the best attraction effect solution is necessarily strictly better than the *MR* solution. For that, let $\gamma^* < \bar{\gamma}$. By construction there exists $\hat{\gamma} \in (\gamma^*, \bar{\gamma}]$ such that using the attraction effect is strictly better than the *MR* solution. But then it is clear that this same solution would also be strictly better than the *MR* solution for a smaller γ. ‖

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