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Revealed (P)Reference Theory[†]

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This paper develops axiomatically a revealed preference theory of reference-dependent choice behavior. Instead of taking the reference for an agent as exogenously given in the description of a choice problem, we suitably relax the Weak Axiom of Revealed Preference to obtain, endogenously, the existence of reference alternatives as well as the structure of choice behavior conditional on those alternatives. We show how this model captures some well-known choice patterns such as the attraction effect. (*JEL* D11, D81)

The canonical model of rational choice maintains that (i) an individual has a well-defined manner of ranking alternatives according to their desirability (independently of any particular choice problem that she might face), and (ii) among *any* collection of feasible alternatives, she chooses the item that she ranks the highest. Despite its various advantages, among which are its unifying structure, universal applicability, tractability, and predictive abilities, this model has long been scrutinized on the basis of its descriptive weaknesses. Indeed, experimental and market evidence points persistently to certain types of choice behavior that are inconsistent with the premises of rational choice theory—human decision processes appear to be more intricate than this theory allows for.[1] In particular, a large amount of evidence suggests that choices are often *reference dependent*: the presence of certain types of options may affect the choice behavior.[2]

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[1] The experimental and market literature that documents violations of the standard rationality paradigm is too extensive to be discussed here: see Camerer (1995) and Rabin (1998) for surveys.

[2] Needless to say, "reference dependence" is a multifarious concept—a "reference" may take the form of one's unattainable aspiration, or the form of an attainable alternative that is desirable according to a social norm. As it will become clear shortly, we focus here on a notion of "reference" which relates to undesirable choice prospects that, when attainable, alter the decision maker's views about the relative appeal of *other* feasible choice options.

In many cases the role of reference point is taken by one specific alternative which can be exogenously identified, such as the status quo choice, the endowment, the default option, etc. At the same time, there is now good evidence that suggests that reference dependence may appear also in choice situations in which no alternative is predesignated as a reference option: a seemingly ordinary feasible choice item may act as a reference for a decision-maker, affecting her choice behavior. While the behavior of such an agent may seem to an outsider hard to "justify," she may actually be acting in a predictable manner relative to her subjectively determined, or endogenous, reference point. A prominent example of this behavior is the *attraction effect* phenomenon (also known as the *asymmetric dominance effect*). Discovered first by Huber, Payne, and Puto (1982), this effect may be described as the phenomenon in which, given a choice set of two feasible alternatives, the addition of a third alternative that is clearly inferior (or else strictly dominated) to *one* of the existing alternatives, but not to the other, may induce a shift of preference toward the item that dominates the new alternative. To illustrate, consider two alternatives, x and y, in a world in which each alternative is characterized by exactly two attributes (such as price and quality). Suppose, as shown in Figure 1, y is better (resp., worse) than x relative to the second (resp., first) attribute. Suppose also that the agent chooses y when only x and y are available. Now suppose a third alternative z becomes feasible; this alternative is inferior to x relative to *both* attributes, but it is still better than y with respect to the first attribute (Figure 1). The attraction effect phenomenon corresponds to the situation in which the agent chooses x from the set $\{x, y, z\}$, violating the standard formulation of rationality. The idea is that, somehow, the asymmetrically dominated alternative z acts as a reference for the agent in the problem $\{x, y, z\}$, making the choice prospect that is unambiguously better than z, namely x, more attractive than it is in the absence of z. Strong evidence of this phenomenon has been found in many studies in psychology and marketing, in different contexts,[3] and in both laboratory and field experiments.[4]

Despite the abundance of evidence for its presence, and its obvious importance for marketing, the consequences of such reference dependent choice behavior are scarcely explored in economic contexts. One difficulty that may be responsible for this is that the example depicted in Figure 1 is only suggestive. It does not tell us exactly how we may model the choice behavior of the agent across various choice problems that she may face. In particular, it is not at all clear how we should think about the relation between referential alternatives across related, for instance,

[3] The attraction effect is demonstrated in the contexts of choice over political candidates (O'Curry and Pitts 1995), choice over risky alternatives (Wedell 1991 and Herne 1997), medical decision-making (Schwartz and Chapman 1999; and Redelmeier and Shafir 1995), investment decisions (Schwarzkopf 2003), job candidate evaluation (Highhouse 1996; Slaughter, Sinar, and Highhouse 1999; and Slaughter 2007), and contingent evaluation of environmental goods (Bateman, Munro, and Poe 2008). While most of the experimental findings in this area are through questionnaire studies, some authors have confirmed the attraction effect also through experiments with incentives (Simonson and Tversky 1992; and Herne 1999).

[4] Doyle et al. (1999) ran a field experiment in a local grocery store. First, the authors recorded the weekly sales of the Brands X and Y in the store, and observed that Brand X had gotten 19 percent of the sales in a given week, and Y the rest, even though Brand X was cheaper. Doyle et al. then introduced a third Brand Z to the supermarket, which was identical to Brand X in all attributes (including the price) except that the size of Brand Z was visibly smaller. The idea is that Brand Z was asymmetrically dominated; it was dominated by X but not by Y. In accordance with the attraction effect, the authors observed in the following week that the sales of Brand X had increased to 33 percent (while nobody had bought Brand Z).

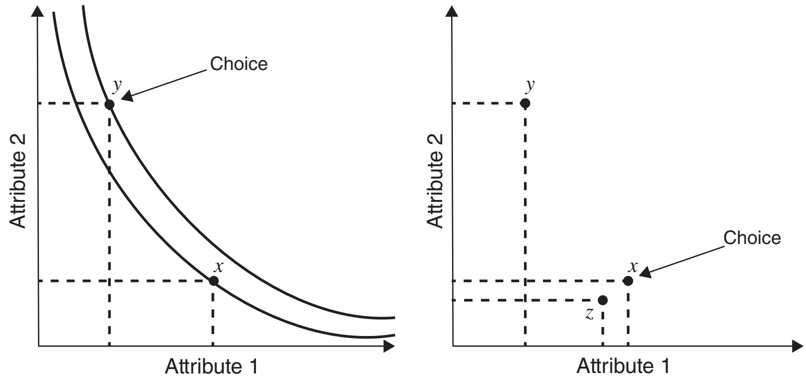

FIGURE 1

nested, choice problems. Also unclear is how to model choices when there is more than one potential reference alternative in a feasible set. (For instance, suppose the choice problem was actually $\{x, y, z, w\}$ in the example above, where w is an alternative that is dominated by y, but not x.) In addition, much of economic theory is built on the premise of preference maximizing behavior, and it is not clear if, and how, one may accommodate the reference dependent behavior of the form above within the rational choice paradigm without deserting this premise entirely. Put differently, it is not evident to what extent such behavior can be reconciled with the standard rational choice model, and whether or not it can be realized within a choice model that nevertheless has a decent predictive power.

We should also note that there is an issue with modeling the attraction effect phenomenon within the realm of revealed preference theory. The illustration given in Figure 1, which is prototypical of the related experimental literature, presumes that alternatives are characterized by a set of exogenously given attributes. While in many applications this assumption may be suitable, it nevertheless precludes the possibility of detecting the attraction effect in environments where the attributes of alternatives that are relevant for choice are private to the decision maker. For instance, suppose x, y, and z are political candidates, and we have observed that an agent votes for y over x, and yet she votes for x when the candidate set is $\{x, y, z\}$. In this case, because we do not know which attributes of the candidates are the relevant ones for the voter, it is not at all obvious if this voting behavior is a manifestation of the attraction effect phenomenon, or if it points to another type of departure from the rational choice paradigm.

Motivated by these observations, we develop in this paper a revealed preference approach toward modeling the attraction effect phenomenon, thereby deriving a model of reference dependent choice with endogenously determined references. Our approach deviates from the classical rational choice theory in a parsimonious manner.[5] In particular, unlike almost the entirety of the related literature, we presume

[5] There are some other models in the literature that aim to account for the attraction effect phenomenon. These models and the related literature are discussed in Section IIIA.

no special structure about the choice alternatives.[6] Rather, as it is standard in choice theory, the primitives of our construction are a collection of feasible sets of alternatives and a choice correspondence defined on this collection. We study choice correspondences that violate the classical formulation of rationality, namely, the *Weak Axiom of Revealed Preference* (WARP), but we focus only on those violations that could be ascribed to the presence of a certain form of (endogenous) reference effect. To this end, we attempt to extend the classical theory of revealed preference to allow not only for revealed *preferences*, but also for revealed *references*. To do this, we first identify two simple rationality conditions on one's choice behavior (Section IB) which should be satisfied by a rational decision maker even if this person may be vulnerable to a phenomenon like the attraction effect. Second, we define two distinct ways of "being a reference" in terms of the choice behavior of the agent alone (Section IC), and then we use these definitions to formulate two revealed preference conditions that introduce some discipline into referential considerations of the individual (Section ID).

In Section IIA, we determine the exact structure of the reference dependent choice model which is consistent with these behavioral postulates. This model has three components: a *utility function* over alternatives; a *reference map* $\mathbf{r}$ that maps any given feasible set S to either a reference alternative $\mathbf{r}(S)$ in that set, or to nothing; and a collection of real functions $\mathcal{U}$ over alternatives which can be seen as the set of attributes the individual deems relevant when making a decision in the presence of a reference point. (The multi-attribute specification of a choice alternative is thus not assumed, but derived, within our approach.) The choice of the agent proceeds as follows. In a given feasible set S she may or may not single out an alternative as a "reference point." If she does not, her behavior is entirely standard: she chooses any one item in S that maximizes her utility function. If, however, she identifies a reference alternative in S—in this case we have $\mathbf{r}(S) \in S$ but $\mathbf{r}(S)$ cannot be a choice from S—rather than maximizing her utility function over the entire S, she focuses only on those members of S that dominate her reference point for *all* attributes in $\mathcal{U}$, that is, only on the elements w in S such that $U(\omega) \geq U(\mathbf{r}(S))$ for every $U \in \mathcal{U}$. The reference point $\mathbf{r}(S)$ defines an "attraction region," composed of the elements that dominate it with respect to all attributes, as if the reference point projects a "spot-light" on a specific subset of the available options. Under this psychological constraint, the agent acts rationally, that is, she finalizes her choice by maximizing her utility function over these options.[7] In Sections IIB and IIC, we study the basic properties of this model, and discuss to what extent its ingredients can be determined uniquely from choice data.

While we do not pursue this direction in the present paper, our model, and its potential variants, can be used, for example, to study the optimal choices of a firm who wishes to exploit the attraction effect in a market environment, and hence to assess the implications of this phenomenon for market segmentation; or to understand implications of the attraction effect for the determination of political candidates and platforms, presentation of portfolios, etc. In Section III, we make

[6] In particular, our model allows for, but does not necessitate, describing alternatives in terms of a given set of attributes. For example, the alternatives could simply be described as objects such as "ice cream," "luxury car A," "presidential candidate B," and so on.

[7] It is easy to see how this model allows for the attraction effect. For example, the model would "explain" the behavior in Figure 1 by saying that it is "as if" the utility of y were higher than that of x—thus y is chosen in $\{x, y\}$—but the agent views z as a reference point in $\{x, y, z\}$, and the attraction region of z includes x but not y.

note of some such applications briefly, and discuss at length the relation of our work to the related literature. The paper concludes with an Appendix that contains the proofs of our main results.

I. Reference-Dependent Choice

A. *Preliminaries*

We work with an arbitrarily fixed separable metric space X, which is thought of as the universal set of available alternatives. We let $\mathfrak{X}$ stand for the set of all nonempty compact subsets of X. The elements of $\mathfrak{X}$ are viewed as feasible sets that a decision maker may need to choose an alternative from; they are henceforth referred to as *choice problems*.

A set-valued map $\mathbf{c} : \mathfrak{X} \rightrightarrows X$ is said to be a choice correspondence on $\mathfrak{X}$ if $\emptyset \neq \mathbf{c}(S) \subseteq S$ for every $S \in \mathfrak{X}$. (If $\mathbf{c}$ is single-valued, we consider it as a function from $\mathfrak{X}$ into X, and refer to it as a choice function on $\mathfrak{X}$.)[8] We say that this correspondence is continuous if for any two convergent sequences (x_m) and (y_m) in X, we have $\lim x_m \in \mathbf{c}\{\lim x_m, \lim y_m\}$ provided that $x^m \in \mathbf{c}\{x^m, y^m\}$ for each m. When X is finite, every choice correspondence on $\mathfrak{X}$ is continuous.

B. *Procedural Rationality Properties of Choice*

The classical assumption of rational choice theory is the so-called *Weak Axiom of Revealed Preference*. Following the formulation of Arrow (1959), we state this property as follows:

Weak Axiom of Revealed Preference (*WARP*).—For every $S \in \mathfrak{X}$ and $T \subseteq S$ with $\mathbf{c}(S) \cap T \neq \emptyset$, we have $\mathbf{c}(S) \cap T = \mathbf{c}(T)$.

The *fundamental theorem of revealed preference* says that for any choice correspondence $\mathbf{c}$ on $\mathfrak{X}$ that satisfies WARP there exists a complete preorder (that is, a complete and transitive binary relation) on X such that, for any $S \in \mathfrak{X}$, the choices from S are the maximum elements in S according to that preorder. Put differently, WARP allows us to view the choices of an individual "as if" they stem from the maximization of a complete preference relation, which is the key feature of rational choice theory.

The primary objective of the present paper is to weaken WARP in order to allow for choice procedures that may depend on reference points that an agent may identify in some choice situations. However, the procedures we wish to consider are to be rational in the absence of referential considerations. Furthermore, loosely speaking, by a "reference" in a choice problem, what we mean is a feasible (but not desirable) alternative in that problem whose presence affects the comparative appeal of other feasible alternatives. From this point of view, pairwise choice situations are special, because in such problems there is no room for a third alternative to act as a reference in this manner. Consequently, the following implication of WARP, namely,

[8] For enumerated finite sets such as $\{x_1, \ldots, x_k\}$, we write $\mathbf{c}\{x_1, \ldots, x_k\}$ instead of $\mathbf{c}(\{x_1, \ldots, x_k\})$ for simplicity. Similarly, if $\mathbf{c}$ is a choice function, we write $x = \mathbf{c}(S)$ instead of $\{x\} = \mathbf{c}(S)$ for any $S \in \mathfrak{X}$.

that one's choices across pairwise choice problems do not exhibit cycles, is likely to remain valid for an interesting class of reference dependent choice correspondences.

The No-Cycle Condition (*No-Cycle*).—For every $x, y, z \in X$, if $x \in \mathbf{c}\{x, y\}$ and $y \in \mathbf{c}\{y, z\}$, then $x \in \mathbf{c}\{x, z\}$.

It is important to note that the No-Cycle Condition separates our analysis from that of most of the literature on violations of WARP, where the presence of cycles from sets of two elements is an essential component of the analysis.[9,10]

Another important implication of WARP concerns the possibility of indifference. Consider the case in which $\{x, y\} \subseteq \mathbf{c}(S)$ for some $S \in \mathfrak{X}$. This could happen for two reasons. First, when the agent is actually indifferent between x and y, that is, when $\{x, y\} = \mathbf{c}\{x, y\}$. The second is when the agent prefers one of the two elements to the other, say x to y, but there is some third element in S which leads the agent to choose y *as well*. In this latter case the reference effect acts by increasing the appeal of y "just enough" to make it indifferent to x. The next postulate rules out this case, thereby imposing that reference effects occur in a discrete, procedural manner—either they come into play, leading the agent to choose one option, or they do not, and the agent chooses the same option she would choose from doubletons.

Rationality of Indifference (*RI*).—For every $x, y \in X$, if $\{x, y\} \subseteq \mathbf{c}(S)$ for some $S \in \mathfrak{X}$, then $\{x, y\} = \mathbf{c}\{x, y\}$.

Notice that if one works with choice *functions*, instead of correspondences, this property is automatically satisfied. Nevertheless, in general, it forces one to look at the notion of a "reference" in a particular (procedural) manner. Relaxing this axiom is likely to allow for choices to depend on references in a more continuous manner than we consider here, but we do not pursue this route in this paper.

C. *Revealed and Potential References*

To extend the classical model of rational choice to incorporate endogenous reference dependence, we need to consider relaxations of WARP that go beyond the previous two axioms. To this end, we next introduce two notions that will help deducing the role of reference alternatives (if any) from the choice behavior of a decision maker. The analysis parallels how one may deduce the preferences of an individual from her choice behavior in revealed preference theory. Consider a pair of alternatives x and z in X, and suppose there is some $y \in X$ such that x is not chosen over y in the pairwise context, that is, $\{y\} = \mathbf{c}\{x, y\}$. However, suppose we observe

[9] This also highlights how the notion of "reference-dependence" we consider here is not related to, say, the status quo bias phenomenon. The latter notion would necessitate a default option, when acting as a reference, to be of additional appeal, and to continue acting as such in dichotomous choice problems as well. To reiterate, by a "reference alternative" in this paper, we mean a feasible alternative that does not attract the decision maker to itself, but rather that alters the relative desirability of other feasible alternatives.

[10] In a recent study, Manzini and Mariotti (2010) tested experimentally how people violate WARP, and classified the violations of WARP in two categories: *pairwise inconsistencies*—choices that violate the No Cycle Condition—and *menu effects*—choices from doubletons do not induce the choices from larger menus. Their conclusion was that the latter are the main reason for violations of WARP.

that x is deemed choosable in $\{x, y, z\}$, that is, $x \in \mathbf{c}\{x, y, z\}$. In other words, x alone is not able to "beat" y, but it does so with the "help" of z. Similarly, if y is deemed choosable from $\{x, y\}$, that is, $y \in \mathbf{c}\{x, y\}$, while x, but not y, is deemed choosable from $\{x, y, z\}$, that is $\{x, y\} \cap \mathbf{c}\{x, y, z\} = \{x\}$, the appeal of x against y seems somewhat "enhanced" when z is also present in the choice problem. This prompts the following definition.

DEFINITION 1: *Let* $\mathbf{c}$ *be a choice correspondence on* $\mathfrak{X}$ *and* $x, z \in X$. *We say that* z *is a* revealed reference *for* x *relative to* $\mathbf{c}$ (*or simply*, *a* revealed $\mathbf{c}$-reference *for* x) *if there is an alternative* $y \in X$ *such that either* (*i*) $x \in \mathbf{c}\{x, y, z\} \backslash \mathbf{c}\{x, y\}$ *or* (*ii*) $y \in \mathbf{c}\{x, y\}$ *and* $\{x, y\} \cap \mathbf{c}\{x, y, z\} = \{x\}$.[11]

This is, however, not the only way we can think of conceptualizing reference alternatives from the choice behavior. Indeed, the concept of a revealed reference is somewhat demanding, for it requires that there be a choice problem from which one element is actually *shown* to "help" the other to be chosen in a way that generates a violation of WARP. One can also think of a related notion according to which the introduction of some alternative z in a feasible set does not "reduce the appeal of x" even though it may not ensure x to be chosen. It is as if z does not really "help" x, but it does not "help" anything else *against* x either. More precisely, if x was chosen against some y when z was not present, then x must continue to be chosen when z is added to $\{x, y\}$, unless z is the unique choice in the new choice problem. Similarly, if y is not chosen against x, then the addition of z should not render y choosable against x. This leads us to the following second formulation of referential alternatives.

DEFINITION 2: *Let* $\mathbf{c}$ *be a choice correspondence on* $\mathfrak{X}$ *and* $x, z \in X$. *We say that* z *is a* potential reference *for* x *relative to* $\mathbf{c}$ (*or simply*, *a* potential $\mathbf{c}$-reference *for* x) *if*, *for every* $y \in X$ *such that* $\mathbf{c}\{x, y, z\} \neq \{z\}$,

$$x \in \mathbf{c}\{x, y\} \textit{ implies } x \in \mathbf{c}\{x, y, z\} \quad \textit{and} \quad y \notin \mathbf{c}\{x, y\} \textit{ implies } y \notin \mathbf{c}\{x, y, z\}.^{12}$$

By contrast to the notion of revealed reference, the notion of potential reference is a fairly weak one. For example, for a rational decision maker (that is, when $\mathbf{c}$ satisfies WARP), z is a potential $\mathbf{c}$-reference for x for any alternatives x and z in X. On the other hand, for such a choice correspondence $\mathbf{c}$, no element of X qualifies as a revealed $\mathbf{c}$-reference for any alternative in X.

D. *Reference-Dependence Properties of Choice*

The notions of potential and revealed references can be given substance only if one is prepared to make some assumptions on the involved choice behavior that would link these concepts to each other. In particular, for an arbitrary choice

[11] In the case of a choice *function* $\mathbf{c}$, this definition simplifies to say that z is a revealed $\mathbf{c}$-reference for x if and only if there is a $y \in X$ such that $y = \mathbf{c}\{x, y\}$ and $x = \mathbf{c}\{x, y, z\}$.

[12] In the case of a choice *function* $\mathbf{c}$, an alternative z is a potential $\mathbf{c}$-reference for x if and only if, for every y in X, $x = \mathbf{c}\{x, y\}$ and $z \neq \mathbf{c}\{x, y, z\}$ imply $x = \mathbf{c}\{x, y, z\}$.

correspondence, there is no a priori reason for a revealed reference to act also as a potential reference, which goes against the motivation behind the formulation of these two concepts. We thus now turn to restrict the way reference effects can take place by connecting the notions of revealed and potential reference.

Consider finitely many alternatives $x_1, \ldots, x_n$ (with $n \geq 2$) such that x_1 is a revealed $\mathbf{c}$-reference for x_2, x_2 is a revealed $\mathbf{c}$-reference for $x_3, \ldots,$ and x_{n-1} is a revealed $\mathbf{c}$-reference for x_n. In this instance, one may at first be tempted to require x_1 to be a revealed $\mathbf{c}$-reference for x_n. This, however, may be too demanding. After all, recall that for x_1 to be a revealed $\mathbf{c}$-reference for x_n, it must "help" it generating a violation of WARP. But it may very well be that the choice of x_n may never require the "help" of x_1 for it is already a rather desirable alternative. It stands to reason, however, that x_1 would never "help" an alternative "against" x_n, for this would point to an inconsistency in the way references arise for the decision maker. To avoid this sort of an inconsistency, x_1 must at least be a potential $\mathbf{c}$-reference for x_n.[13] This prompts the following postulate.

Reference Acyclicity (*RA*).—For any integer $n \geq 2$ and $x_1, \ldots, x_n \in X$, if x_i is a revealed $\mathbf{c}$-reference for x_{i+1} for each $i = 1, \ldots, n-1$, then x_1 is a potential $\mathbf{c}$-reference for x_n.

We now come to our most important behavioral postulate, namely, *Reference Consistency*. To formulate this property, let us recall that a collection $\mathcal{T}$ of subsets of a set S is said to be a cover of S if the union of the members of $\mathcal{T}$ equals S, that is, $\cup \{T : T \in \mathcal{T}\} = S$. (Notice that $\mathcal{T}$ is not necessarily a partition of S; its members are allowed to overlap.) In turn, given a choice correspondence $\mathbf{c}$ on $\mathfrak{X}$, and a choice problem $S \in \mathfrak{X}$, we say that a collection $\mathcal{T} \subseteq 2^S \cap \mathfrak{X}$ is a $\mathbf{c}$-cover of S if $\mathcal{T}$ is a cover of S such that each member of $\mathcal{T}$ contains at least one choice from S with respect to $\mathbf{c}$, that is, $\mathbf{c}(S) \cap T \neq \emptyset$ for each $T \in \mathcal{T}$. Under WARP, if $\mathcal{T}$ is a $\mathbf{c}$-cover of S, then we must have $\mathbf{c}(T) = \mathbf{c}(S) \cap T$ for *all* $T \in \mathcal{T}$. Our next axiom weakens this requirement by postulating that, for any choice problem S, WARP should hold for *at least one* member of any $\mathbf{c}$-cover of S that includes at least one doubleton set.

Reference Consistency (*RCon*).—For any $S \in \mathfrak{X}$ and $\mathbf{c}$-cover $\mathcal{T}$ of S with $|T| = 2$ for some $T \in \mathcal{T}$, we have $\mathbf{c}(T') = \mathbf{c}(S) \cap T'$ for some $T' \in \mathcal{T}$.

RCon is a consistency condition that aims at restricting the violations of WARP only to the phenomenon of reference-dependence (instead of other types of menu-dependent behavior). To illustrate the content of this postulate, suppose for a moment that we could observe not only the agent's choices from a set S, but also her reference point, say, $z \in S$. Now consider a subset T of S that contains not only some of the chosen elements ($\mathbf{c}(S) \cap T \neq \emptyset$), but also the reference point ($z \in T$). This means that even though T is a subset of S, it preserves the "key" components of choice from S, and it seems reasonable that the reference point

[13] Implicit in this formulation is the idea that "being a reference" is an all-or-nothing phenomenon. Loosely speaking, if z is a reference for both x and y, we rule out the possibility that z is "more of a reference" for x than for y. On the one hand, this simplifies the revealed preference theory that we are about to sketch. On the other, we are not aware of any evidence that motivates the modeling of the notion of "being a reference" as a graded phenomenon.

should retain its power in this smaller set T as well, leading the agent once again to those choices (from S) that are still available in T, that is, $\mathbf{c}(T) = \mathbf{c}(S) \cap T$.

Of course, in actuality, we are not privy to the reference point of the agent in S, that is, z is unobservable. Moreover, it is possible that the agent may simply be maximizing her utility in S without recourse to any referential considerations. Consequently, we cannot write down an axiom that corresponds to the discussion above directly. Instead, by means of RCon, we impose an implication of this sort of behavior. To wit, consider a $\mathbf{c}$-cover $\mathcal{T}$ of a set S such that $|T| = 2$ for some $T \in \mathcal{T}$. Suppose first that the agent does not use a reference in S; her choices from S are simply the best ones according to some reference-free ranking (utility) of the alternatives in S. But since $|T| = 2$, then it corresponds to a pairwise choice situation, and she does not use a reference in T either. Consequently, rationality (in the garb of WARP) demands that she chooses from T also by maximization of utility, that is, $\mathbf{c}(T) = \mathbf{c}(S) \cap T$, and RCon is satisfied. Now suppose, instead, that the decision maker does use a reference alternative in making her choices from S. Even though we do not observe this reference point, we know that it must belong to *at least one* of the elements of $\mathcal{T}$, say T', since $\mathcal{T}$ is a $\mathbf{c}$-cover of S. But then, T' contains both the reference point of S and at least one alternative chosen from S. Then, just as in the previous paragraph, rationality (in the garb of WARP) requires that $\mathbf{c}(T') = \mathbf{c}(S) \cap T'$, and this is precisely what RCon posits.

Remark 1: In the case where X is finite (or more generally, when $\mathfrak{X}$ consists of all nonempty finite subsets of X), the statement of RCon can be simplified considerably, for in that case we can work with $\mathbf{c}$-covers composed of only three sets. That is, in that case, $\mathbf{c}$ satisfies RCon if and only if, for any $S \in \mathfrak{X}$ and any three subsets T_1, T_2, and T_3 of S, with at least one of which being a doubleton, we have $\mathbf{c}(T_j) = \mathbf{c}(S) \cap T_j$ for some j, provided that $T_1 \cup T_2 \cup T_3 = S$ and $\mathbf{c}(S) \cap T_i \neq \emptyset$ for each i. This observation, which may be proved by induction, makes RCon relatively easy to test in experiments.

We conclude this section by noting that each of the four axioms we have introduced above corresponds to a weakening of WARP. Indeed, while No-Cycle, RCon, and RI are trivially weaker than WARP, the RA property is weaker than WARP because, when $\mathbf{c}$ satisfies WARP, there does not exist an alternative that can act as a revealed $\mathbf{c}$-reference. In the Appendix we prove that these four behavioral postulates constitute a logically independent set of axioms.

II. Representation Theorem

A. *The Reference-Dependent Choice Model*

We now turn to discuss our main result, a representation for choice correspondences that satisfy the four behavioral properties discussed above. Let us fix a symbol $\Diamond$ (formally, a generic object that does not belong to X), and for a given set $\mathcal{U}$ of functions from X to $\mathbb{R}$, define $\mathcal{U}^{\uparrow}(x)$ as the set of elements that dominate the alternative $x \in X$ for each function in the set, that is,

$$\mathcal{U}^{\uparrow}(x) := \{y \in X : U(y) \geq U(x) \text{ for every } U \in \mathcal{U}\},$$

for any $x \in X$.

We are now ready to state our main result.

THEOREM 1: *A continuous choice correspondence* $\mathbf{c}$ *on* $\mathfrak{X}$ *satisfies No-Cycle, RA, RCon, and RI if and only if there exist a continuous function* $u : X \to \mathbb{R}$, *a nonempty set* $\mathcal{U}$ *of real maps on X, a function* $\mathbf{r} : \mathfrak{X} \to X \cup \{\lozenge\}$ *with* $\mathbf{r}(S) \in S \backslash \mathbf{c}(S)$ *whenever* $\mathbf{r}(S) \neq \lozenge$, *such that, for every* $S \in \mathfrak{X}$,

(*i*) *if* $\mathbf{r}(S) = \lozenge$,

$$\mathbf{c}(S) = \arg\max u(S); \tag{1}$$

(*ii*) *if* $\mathbf{r}(S) \neq \lozenge$,

$$\mathbf{c}(S) = \arg\max u(S \cap \mathcal{U}^{\uparrow}(\mathbf{r}(S))); \tag{2}$$

(*iii*) *for any* $T \subseteq S$ *such that* $\mathbf{r}(S) \in T$ *and* $\mathbf{c}(S) \cap T \neq \emptyset$, *we have* $\mathbf{r}(T) \neq \lozenge$ *and*

$$\mathbf{c}(T) = \arg\max u(T \cap \mathcal{U}^{\uparrow}(\mathbf{r}(S))). \tag{3}$$

Let $\langle u, \mathbf{r}, \mathcal{U} \rangle$ be an ordered triplet where u is a continuous real map on X, $\mathbf{r} : \mathfrak{X} \to X \cup \{\lozenge\}$ is a function, and $\mathcal{U}$ is a nonempty set of real maps on X. We say that this triplet represents a choice correspondence $\mathbf{c}$ on $\mathfrak{X}$ if (1) holds for each $S \in \mathfrak{X}$ with $\mathbf{r}(S) = \lozenge$ and (2) holds for each $S \in \mathfrak{X}$ with $\mathbf{r}(S) \neq \lozenge$. In addition, if $\mathbf{r}(S) \in S \backslash \mathbf{c}(S)$ whenever $\mathbf{r}(S) \neq \lozenge$, and $\langle u, \mathbf{r}, \mathcal{U} \rangle$ satisfies the property (*iii*) of Theorem 1 for the choice correspondence $\mathbf{c}$ that it represents, we say that $\langle u, r, \mathcal{U} \rangle$ is a reference dependent choice model on $\mathfrak{X}$.

In words, Theorem 1 says that a continuous choice correspondence $\mathbf{c}$ on $\mathfrak{X}$ satisfies the four behavioral properties we have discussed above if and only if it can be represented by a reference dependent choice model $\langle u, \mathbf{r}, \mathcal{U} \rangle$ on $\mathfrak{X}$. One interpretation of the model is as follows. First, u is viewed as a standard utility function for the decision-maker, free of any referential considerations. Then, the map $\mathbf{r}$, which we refer to as the reference map of the model, tells us whether the agent uses a reference point in a given choice problem S or not. Finally, $\mathcal{U}$ can be seen as the set of attributes of the objects that the agent deems relevant for choice.[14] In turn, the decision making of the individual follows the following procedure. For any choice problem $S \in \mathfrak{X}$, the agent either evaluates what to choose in a reference-independent manner, or identifies a reference point in S and uses it to finalize her choice. In the former case we have $\mathbf{r}(S) = \lozenge$ and the agent simply maximizes her reference-free utility, that is, $\mathbf{c}(S) = \arg\max u(S)$, in concert with the standard theory

[14] When X is finite, $\mathcal{U}$ can be taken as a finite collection in Theorem 1.

of rational choice.[15] In the latter case, the reference point in S corresponds to the alternative $\mathbf{r}(S)$. This alternative is not chosen in S. It is rather used to identify those alternatives of S to which the agent is "mentally attracted." These alternatives are precisely the ones in S which dominate $\mathbf{r}(S)$ with respect to *all* the attributes that are deemed relevant for choice, that is, the elements x of S such that $v(x) \geq v(\mathbf{r}(S))$ for every v in $\mathcal{U}$. It is *as if* the reference point generates a *mental constraint* for the agent, leading her to focus only on those elements that are better than $\mathbf{r}(S)$ in every relevant aspect. Within this constraint, the agent acts fully rationally, and solves her problem by maximizing u, that is, $\mathbf{c}(S) = \arg\max u(S \cap \mathcal{U}^{\uparrow}(\mathbf{r}(S)))$.

Finally, (3) imposes some consistency between the references and choices of an individual from nested sets. Take a problem $S \in \mathfrak{X}$ in which the agent uses a reference point $\mathbf{r}(S) \neq \Diamond$, and consider another choice problem $T \subseteq S$ which contains some of the choices from S (that is, $\mathbf{c}(S) \cap T \neq \emptyset$) as well as the reference used in S (that is, $\mathbf{r}(S) \in T$). It may first seem appealing to presume that the agent would not change her referential assessment in the contexts of S and T, thereby asking for the property $\mathbf{r}(T) = \mathbf{r}(S)$. However, such a requirement is simply too demanding in general. To wit, consider two sets S and S', and suppose that they have different reference points, that is, $\mathbf{r}(S) \neq \mathbf{r}(S')$. Now consider a subset T of both S and S', and suppose that T contains both $\mathbf{r}(S)$ and $\mathbf{r}(S')$ as well as some of the choices from S and S'. This appears to be a rather natural situation, and yet it would be impossible under the condition above: it would require $\mathbf{r}(T) = \mathbf{r}(S)$ and $\mathbf{r}(T) = \mathbf{r}(S')$, which contradicts $\mathbf{r}(S) \neq \mathbf{r}(S')$.

What one can guarantee, however, is that for two choice problems S and T, with $T \subseteq S$ and $\mathbf{c}(S) \cap T \neq \emptyset$, the reference points of S and T, even if they are distinct, would depict the same influence on one's choice behavior in the sense that they lead to consistent choices from S and T as rationality demands. This is exactly what is required by (3): when T is nested in S, and both a choice from S and the reference of S remain feasible in T, then, even if a different reference point could be used in T, the agent's choices would be identical to the ones she would have picked if she used $\mathbf{r}(S)$ as the reference point in T. In turn, this guarantees that, if a reference dependent choice model $\langle u, \mathbf{r}, \mathcal{U} \rangle$ represents $\mathbf{c}$, then

$$\mathbf{r}(S) \in T \subseteq S \text{ and } \mathbf{c}(S) \cap T \neq \emptyset \quad \text{imply} \quad \mathbf{c}(T) = \mathbf{c}(S) \cap T$$

for any $S, T \in \mathfrak{X}$.

In sum, the reference dependent choice model derived in Theorem 1 portraits an individual as behaving exactly as prescribed by the attraction effect phenomenon by positing that her attention is "attracted" to those elements that dominate one alternative (with respect to each relevant criteria). Thus, this representation naturally allows for the attraction effect. However, the choice behavior this model corresponds to is more general, for Theorem 1 obtains *endogenously*, and on the basis of purely behavioral postulates, not only the reference points of the decision-maker, but also her *subjective* attributes in terms of which she evaluates her

[15] The standard theory thus corresponds to the special case of our model in which $\mathbf{r}(S) = \Diamond$ for all $S \in \mathfrak{X}$.

choice prospects. Theorem 1 thus identifies how one can check for the presence of an attraction effect type behavior also in contexts where attributes of the choice prospects are not explicitly given, and provides a choice model that accounts for this effect across any choice domain.

B. *Properties of Reference-Dependent Choice Models*

Using the terminology of Section IC, we now identify a few properties of choice correspondences that are represented by reference dependent choice models. Suppose S is a choice problem such that $\mathbf{c}(S) \neq \arg\max u(S)$. In this case, the representation in Theorem 1 above maintains that $\mathbf{r}(S)$ is an alternative in S which is not chosen. We now show that this alternative is indeed viewed as a "reference" by $\mathbf{c}$ in the sense that $\mathbf{r}(S)$ is a revealed $\mathbf{c}$-reference for every choice from S, thereby linking our model to the behavioral definition above.

PROPOSITION 1: *Let* $\mathbf{c}$ *be a choice correspondence on* $\mathfrak{X}$ *that is represented by a reference dependent choice model* $\langle u, \mathbf{r}, \mathcal{U}\rangle$. *Let* $S \in \mathfrak{X}$ *be a choice problem with* $\mathbf{c}(S) \neq \arg\max u(S)$ *: Then,* $\mathbf{r}(S)$ *is a revealed* $\mathbf{c}$ *-reference for each* $x \in \mathbf{c}(S)$.

PROOF:

Put $z := \mathbf{r}(S)$. Since $\mathbf{c}(S) \neq \arg\max u(S)$, (2) implies that there is y in $(\arg\max u(S))\backslash\mathbf{c}(S)$. Then, for any $x \in \mathbf{c}(S)$, we may use (3) to find that $y \in \mathbf{c}\{x, y\}$ and $\{x, y\} \cap \mathbf{c}\{x, y, z\} = \{x\}$. This means that z is a revealed $\mathbf{c}$-reference for x. ∎

The next result highlights the connection between reference alternatives and the collection of real maps $\mathcal{U}$ for choice correspondences that are represented by $\langle u, \mathbf{r}, \mathcal{U}\rangle$.

PROPOSITION 2: *Let* $\mathbf{c}$ *be a choice correspondence on* $\mathfrak{X}$ *that is represented by a reference dependent choice model* $\langle u, \mathbf{r}, \mathcal{U}\rangle$. *For every* $x, z \in X$,

(*i*) *if* z *is a revealed* $\mathbf{c}$*-reference for* x, *then* $U(x) \geq U(z)$ *for every* $U \in \mathcal{U}$;

(*ii*) *if* $U(x) \geq U(z)$ *for every* $U \in \mathcal{U}$, *then* z *is a potential* $\mathbf{c}$*-reference for* x.

PROOF:

Take any x and z in X such that z is a revealed $\mathbf{c}$-reference for x. Then, there is a $y \in X$ such that either (i) $x \in \mathbf{c}\{x, y, z\}\backslash\mathbf{c}\{x, y\}$ or (ii) $y \in \mathbf{c}\{x, y\}$ and $\{x, y\} \cap \mathbf{c}\{x, y, z\} = \{x\}$. In either of these cases, it is clear that $\mathbf{c}\{x, y, z\} \neq \arg\max u(\{x, y, z\})$, so $\mathbf{r}\{x, y, z\} \neq \Diamond$, by the representation of $\mathbf{c}$. On the other hand, if either x or y is $\mathbf{r}\{x, y, z\}$, then (3) readily yields a contradiction. Thus: $\mathbf{r}\{x, y, z\} = z$ and the representation now ensures that $U(x) \geq U(z)$ for every $U \in \mathcal{U}$. This proves part (i) of the proposition. The proof of part (ii) is similar. ∎

In words, Proposition 2 says that every alternative attracts the decision maker to those items that it is revealed to be a reference for, and conversely, if an alternative happens to attract the agent to x, then that alternative must at least be a potential reference for x.

Remark 2: An immediate implication of the Reference Acyclicity (RA) axiom is that every revealed **c**-reference is a potential **c**-reference. Given the interpretation of revealed and potential **c**-references, it may be of interest to see how the reference dependent choice model would modify if we were to impose only this latter property instead of RA in Theorem 1. We note here (without proof) that if a continuous choice correspondence **c** on X satisfies No-Cycle, RCon, and RI, and every revealed **c**-reference is a potential **c**-reference, then there exists a continuous real function u, a correspondence $Q : X \cup \{\Diamond\} \rightrightarrows X$ with $Q(\Diamond) = X$, and a function $\mathbf{r} : \mathfrak{X} \to X \cup \{\Diamond\}$ with $\mathbf{r}(S) \in S \backslash \mathbf{c}(S)$ whenever $\mathbf{r}(S) \neq \Diamond$, such that

$$\mathbf{c}(S) = \arg\max u(S \cap Q(\mathbf{r}(S))) \qquad \text{for each } S \in \mathfrak{X}.$$

Furthermore, for any S and T in $\mathfrak{X}$ such that $T \subseteq S$, $\mathbf{r}(S) \in T$ and $\mathbf{c}(S) \cap T \neq \emptyset$, we have $\mathbf{r}(T) \neq \Diamond$ and

$$\mathbf{c}(T) = \arg\max u(T \cap Q(\mathbf{r}(S))).$$

(The converse of this statement is also valid.) Thus, the "maximization under a constraint (mentally) induced by a reference point" nature of the reference dependent choice model would be retained under the said weakening of RA as well. In particular, the contents of the Propositions 1 and 2 remain valid for this model (with trivial modifications in the statements of these results.) However, within this weaker axiomatic system, we would not be able to describe the values of the mental constraint Q as the set of all alternatives that dominate the associated reference points with respect to a set of (attribute) functions.

C. *Uniqueness of Reference Dependent Choice Models*

Suppose two reference dependent choice models $\langle u_1, \mathbf{r}_1, \mathcal{U}_1 \rangle$ and $\langle u_2, \mathbf{r}_2, \mathcal{U}_2 \rangle$ on $\mathfrak{X}$ are *behaviorally equivalent*, that is, they represent the same choice correspondence on $\mathfrak{X}$. What can we say about the relation between the ingredients of these two models? The answer is straightforward in the case of u_1 and u_2. These functions are continuous, and they represent the same preference relation on X, and hence, they must be continuous and strictly increasing transformations of each other. The situation is more subtle for the reference maps and the sets of attributes, however. We begin with the former.

Example 1 (Non-Uniqueness of Reference Maps): Let $X := \{x, y, z_1, z_2\}$, and consider a choice correspondence **c** on $2^X \backslash \{\emptyset\}$ such that $\mathbf{c}\{x, y\} = \{y\}$ and $\mathbf{c}(S) = \{x\}$ for every non-singleton subset S of X that contains x and is distinct from $\{x, y\}$. A behavior of this kind is typical of the attraction effect phenomenon—compare Figures 1 and 2—and naturally, it can be captured by a reference dependent choice model. Suppose $\langle u, \mathbf{r}, \mathcal{U} \rangle$ is one such model. Then, clearly, $u(y) > u(x) > \max\{u(z_1), u(z_2)\}$ and $\mathbf{r}(S) \in \{z_1, z_2\}$ for every subset S of X that contains x, y and at least one of the alternatives z_1 and z_2. The reference map **r** is, therefore, uniquely

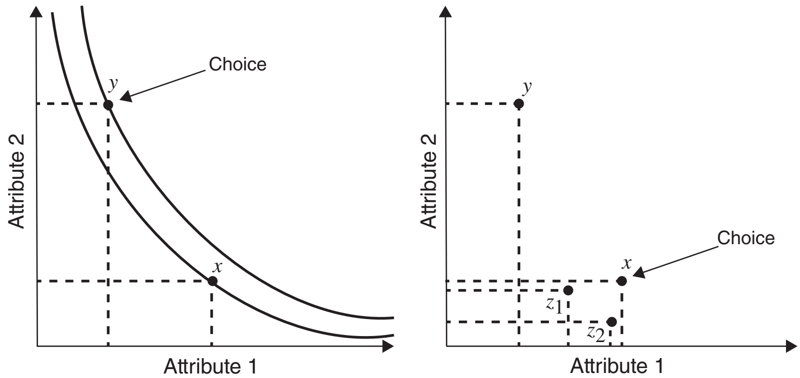

Figure 2

identified at any nonempty proper subset S of X such that $\mathbf{c}(S) \neq \arg\max u(S)$. However, insofar as the choice behavior is concerned, the model cannot possibly distinguish between the referential attributes of z_1 and z_2; putting either $\mathbf{r}(X) = z_1$ or $\mathbf{r}(X) = z_2$ makes no behavioral difference. Put differently, if $\mathbf{r}(X) = z_1$, and $\mathbf{r}'$ is a reference map on $\mathfrak{X}$ that agrees with $\mathbf{r}$ at every nonempty proper subset of X but has $\mathbf{r}'(X) = z_2$, then $\langle u, \mathbf{r}', \mathcal{U}\rangle$ represents $\mathbf{c}$ as well.

This example suggests that there is a sense of "arbitrariness," in the specification of a reference map in general. Consequently, the uniqueness properties of a reference dependent model in terms of its reference map would best be identified by focusing on *all* options that can act as references in a given choice problem simultaneously. To this end, we introduce the notion of a reference *correspondence.*

DEFINITION 3: *Let* $\langle u, \mathbf{r}, \mathcal{U}\rangle$ *be a reference dependent choice model that represents a choice correspondence* $\mathbf{c}$ *on* $\mathfrak{X}$, *and let* $\mathfrak{Y}$ *stand for the set of all* S *in* $\mathfrak{X}$ *such that* $\mathbf{c}(S) \neq \arg\max u(S)$. *We define the* reference correspondence $\mathbf{R} : \mathfrak{Y} \to 2^X$ *associated with this model by taking* $\mathbf{R}(S)$ *as the set of all* z *in* S *such that*

$$\mathbf{c}(T) = \arg\max u(T \cap \mathcal{U}^{\uparrow}(z)) \quad \text{for every } T \in \mathfrak{X} \text{ with } z \in T \subseteq S \text{ and } \mathbf{c}(S) \cap T \neq \emptyset.$$ [16]

In words, given a reference dependent choice model $\langle u, \mathbf{r}, \mathcal{U}\rangle$ that represents a choice correspondence $\mathbf{c}$ on $\mathfrak{X}$, the correspondence $\mathbf{R}$ maps any feasible set S (at which choice is not obtained by maximization of u) to the collection of *all* alternatives that could be used as a reference point in the set S; $\mathbf{r}(S)$ is only one of these alternatives. (For instance, if $\mathbf{c}$ is the choice correspondence of Example 1, we have $\mathbf{R}(X) = \{z_1, z_2\}$.)

Let us now turn to the uniqueness of the collection $\mathcal{U}$ of attributes in a given reference dependent choice model. In that case, the uniqueness issue is not so much about "arbitrariness," but about "redundancy." In effect, the problem is determining when an alternative of higher reference-free utility fails to dominate a reference

[16] In view of Theorem 1, $\mathbf{c}$ plays only an auxiliary role here. That is, $\mathbf{R}$ is determined uniquely once $\langle u, \mathbf{r}, \mathcal{U}\rangle$ is specified; we use the choice correspondence represented by $\langle u, \mathbf{r}, \mathcal{U}\rangle$ in the definition only to simplify the involved notation.

alternative in terms of the given set of all attributes. To clarify this, we will use the following auxiliary definition.

DEFINITION 4: *Let* $\langle u, \mathbf{r}, \mathcal{U} \rangle$ *be a reference dependent choice model on* $\mathfrak{X}$. *We define the correspondence* $\mathcal{R} : X \rightrightarrows X$ *by letting* $\mathcal{R}(z)$ *to stand for the collection of all* $x \in X$ *for which there is a* $y \in X$ *and a* $v \in \mathcal{U}$ *such that* $\mathbf{r}\{x, y, z\} = z$, $u(y) > u(x)$ *and* $v(z) > v(y)$.

It is worth noting that $\mathcal{R}$ has behavioral content. Indeed, given a reference dependent choice model $\langle u, \mathbf{r}, \mathcal{U} \rangle$ that represents $\mathbf{c}$ on $\mathfrak{X}$, it is easily shown that

$$(4) \qquad \mathcal{R}(z) = \{x \in X : z \text{ is a revealed } \mathbf{c}\text{-reference for } x\}, \qquad z \in X.$$

We are now ready to characterize the uniqueness properties of reference dependent choice models.

PROPOSITION 3: *Let* $\mathbf{c}$ *be a choice correspondence on* $\mathfrak{X}$ *that is represented by a reference dependent choice model* $\langle u_1, \mathbf{r}_1, \mathcal{U}_1 \rangle$. *Then, a reference dependent choice model* $\langle u_2, \mathbf{r}_2, \mathcal{U}_2 \rangle$ *represents* $\mathbf{c}$ *if, and only if,* $u_2 = f \circ u_1$ *for some continuous and strictly increasing* $f : u_1(X) \to \mathbb{R}$, $\mathbf{R}_1 = \mathbf{R}_2$, *and* $\mathcal{R}_1 = \mathcal{R}_2$.

In words, the utility functions of two behaviorally equivalent reference dependent choice models must be ordinally equivalent. Moreover, the difference between the collections of (attribute) functions utilized in these two models must not be due to the revealed references they induce. Finally, they must induce the same reference correspondence, which means that the difference between the involved reference maps must be due only to the "arbitrariness" matter we have discussed above. The following result, which follows readily from Proposition 3, aims to drive this point home.

COROLLARY 1: *Let* $\langle u_1, \mathbf{r}_1, \mathcal{U}_1 \rangle$ *and* $\langle u_2, \mathbf{r}_2, \mathcal{U}_2 \rangle$ *be two reference dependent choice models that represent the same choice correspondence* $\mathbf{c}$. *Then, for every* $S \in \mathfrak{X}$ *with* $\mathbf{c}(S) \neq \arg\max u_1(S)$, *we have* $\mathbf{c}(S) = \arg\max u_1(S \cap \mathcal{U}_1^{\uparrow}(\mathbf{r}_2(S)))$.

This result shows that whenever two reference dependent choice models induce the same choice correspondence the reference points they use in any choice problem where the individual does not simply maximize utility are interchangeable. As this fact is independent of the other elements of the representation, we may conclude that the non-uniqueness of the reference map in a reference dependent choice model arises only due to a form of arbitrariness (as in Example 1) which is, behaviorally speaking, inconsequential.

III. On Related Literature and Applications

A. *Literature on Reference Dependent Choice Models*

The notion of "reference dependence" has been extensively investigated in economics. Indeed, there is a sizable literature on modeling choice problems for which reference points are *exogenously* given.[17] By contrast, we study choice situations that do not come with any preassigned reference point, the presence/absence of such options for a decision-maker is determined by observing choices across various situations, very much in the tradition of the revealed preference theory.

To our knowledge, the only other papers that focus on endogenous reference formation are Köszegi and Rabin (2006, 2007). These papers develop a model in which the agent's reference point is determined as her rational expectations about the outcome she will receive given her behavior, which in turn must be optimal in terms of a classical gain-loss utility function conditional on this reference point. This approach differs from ours in several aspects. First, in the Köszegi-Rabin approach one's reference point is a belief about future returns, and hence it is, at least conceptually speaking, not a choice item. And when a lottery corresponding to the reference point does belong to the set, it emerges as a desirable alternative; the individual may well choose this object in a choice problem. By contrast, in our model a reference point in a choice problem, if it arises, is an option that is surely *feasible* in that problem, but one that the agent *will never choose* (as in the attraction effect). Second, the model of Köszegi and Rabin (2006) coincides with that of rational choice if there is no underlying uncertainty, which means that it is not suitable to address the instances of reference dependent behavior, like the attraction effect, in environments in which no uncertainty is present. By contrast, our model allows for uncertainty as a special case, but also addresses reference-dependence under certainty as well.

Recent papers have introduced choice models compatible with the attraction effect phenomenon. de Clippel and Eliaz (2012) axiomatizes a model where one's choices are the cooperative solution of a bargaining problem between two preference relations; this can generate certain instances of the attraction effect (as well as other choice anomalies). Lombardi (2009) introduces a model in which the agent selects the elements that are maximal according to some acyclic binary relation, and then she eliminates from the resulting maximal set those alternatives whose lower contour sets are strictly contained in that of some other maximal alternative. There is one key difference between these models and ours: they are compatible with only a "weak" form of the attraction effect, as they allow for the addition of an asymmetrically dominated option to help the agent decide between two alternatives to which she reveals to be *indifferent* in a pairwise comparison, but none of them allows for this addition to lead the agent to alter her strict ranking of choice items.[18] In particular, if we specialize these models to the case of choice *functions*, none of them remain compatible with the attraction effect. Yet, an overwhelming part of the data on the attraction effect is of the latter form (and is in fact presented in terms of choice functions).

Our choice model is also related to models in which the agent focuses on specific subsets of the available options. Masatlioglu, Nakajima, and Ozbay (2012) present

[17] Among them are the behavioral (loss aversion) models of Kahneman and Tversky (1979) and Tversky and Kahneman (1991), and the axiomatic choice models of Chateauneuf and Wakker (1999); Masatlioglu and Ok (2005, 2014); Sagi (2006); Diecidue and van de Ven (2008); and Ortoleva (2010).

[18] More precisely, all of these models are compatible with the following choice data: $\mathbf{c}\{x,y\} = \{x,y\}$ and $\mathbf{c}\{x,y,z\} = \{y\}$—the introduction of z breaks the indifference between x and y—but none of them is compatible with the choice data $\mathbf{c}\{x,y\} = \{x\}$ and $\mathbf{c}\{x,y,z\} = \{y\}$—the introduction of z leads the agent to alter her choice from x to y.

a related model in which there are finitely many alternatives, choice behavior is modeled through choice functions, and subjects' choice maximizes the utility focusing only on the elements in a subset $\Gamma(S)$ of the choice problem S. They posit that the self-map Γ on $2^X\backslash\{\emptyset\}$ is such that $\Gamma(S) = \Gamma(S\backslash\{x\})$ holds whenever $x \in S\backslash\Gamma(S)$, and refer to any such map as an *attention filter*. If we restrict our attention to the case where X is finite, and work only with a choice function $\mathbf{c}$, then our model can be seen as a special case of that model.[19] In the special case of finite X and choice functions, however, their model is much more general.[20] Cherepanov, Feddersen, and Sandroni (2013) present a model composed of a binary relation $\succeq$ on X and a map ψ such that $\emptyset \neq \psi(S) \subseteq S$ for all choice problems S. The agent's choice is the set of alternatives that maximize $\succeq$ in $\psi(S)$. Assuming a finite set of options, they provide axiomatic foundation for the case in which the agent has a collection Λ of rationales (binary relations on X), and $\psi(S)$ is found as the set of elements of S that are maximal with respect to at least one of the rationales in Λ. While our models are clearly related, they are not nested.[21]

Finally, our model is related to models of choice built on a two-stage decision making procedure. Manzini and Mariotti (2007) and Rubinstein and Salant (2006) axiomatize choice functions that can be represented as if the agent applies two binary relations, one after the other:[22] first, she eliminates all elements in a problem that are not maximal relative to the first relation, and second, among the remaining elements chooses that which is optimal according to the second relation. Despite some similarities, these models study violations of WARP that are complementary to those that we study: the intersection of the class of choice correspondences that we have characterized here and those of Manzini and Mariotti (2007) and Rubinstein and Salant (2006) contains only the classical rational choice model (for choice functions). This is evident from the fact that in our analysis we posit that there cannot be cyclic behavior in choice from doubleton sets—our No-Cycle axiom; but it is well-known that under such provisions, both models reduce to rationality.

We should also note that there are some other approaches toward understanding the attraction effect in the literature. In particular, Kamenica (2008) studies a model in which there is a market with rational consumers some of whom are informed and some of whom are uninformed, with uninformed consumers exhibiting a behavior in equilibrium that conforms with the attraction effect phenomenon. In this model, choice anomalies are not seen as violations of "rationality," but they rather emerge as equilibrium behavior in a specific market environment. It is also worth noting that Natenzon (2014) has recently introduced a model in which the agent gradually learns

[19] Setting $\Gamma(S) := \{\mathbf{c}(S)\} \cup \{x \in X : u(\mathbf{c}(S)) > u(x)\}$, we find that Γ is an attention filter and $\{\mathbf{c}(S)\} = \arg\max u(\Gamma(S))$ for every nonempty $S \subseteq X$.

[20] Lleras et al. (2010) study a related choice model where the defining property of the map Γ is that $x \in \Gamma(S)$ implies $x \in \Gamma(T)$ for every $T \subseteq S$ with $x \in T$. Simple examples would show that the resulting choice model and that of Theorem 1 are not nested.

[21] On the one hand, there are instances that are allowed by the Cherepanov-Feddersen-Sandroni model but not by ours—like cyclic choice in pairwise choice situations. On the other hand, there are behaviors allowed for by our model and not by theirs. For example, let $X := \{x, y, z, w\}$, and consider the choice function $\mathbf{c}$ on $2^X\backslash\{\emptyset\}$ defined by $\mathbf{c}\{x, y\} = \mathbf{c}\{x, z\} = \mathbf{c}\{x, w\} = \mathbf{c}\{x, y, z\} = \mathbf{c}\{x, z, w\} := x$, $\mathbf{c}\{y, z\} = \mathbf{c}\{y, w\} = \mathbf{c}\{x, y, w\} = \mathbf{c}\{y, z, w\} := y$, $\mathbf{c}\{z, w\} := z$ and $\mathbf{c}(X) := x$. This function cannot be represented as in the Cherepanov-Feddersen-Sandroni model, but it can be represented by a reference-dependent choice model.

[22] For papers that work with sequential procedures with more than two stages, see Apestegia and Ballester (2013) and Manzini and Mariotti (2012).

the utility of available alternatives and shows how this can generate, in the context of random choice, the attraction effect when subjects are relatively uninformed. By contrast to Kamenica's work, we regard the attraction effect as a violation of WARP in this paper. And by contrast to Natenzon's work, we study this phenomenon in the context of deterministic choice. Our approach is motivated by the presence of the attraction effect phenomenon in very different sorts of economic environments, as well as in laboratory experiments, where the informational structure does not seem to be the source of the problem.

B. *On Potential Applications*

We have used the notions of "revealed" and "potential" references in this paper in a particular manner (through the axioms RA and RCon). These notions are defined in terms of one's choice behavior in general, and therefore, their use is in no way restricted to the model of Theorem 1. As such, these concepts, or variants of them, may be useful in studying other types of departures from the rational choice paradigm by the revealed preference method.

In turn, the choice model we have introduced here itself could be used to study the implications of the attraction effect in a variety of economic settings, especially when the prospects possess multiple attributes relevant for choice (may they be objective or subjective). For example, in multi-dimensional (spatial) voting problems with at least three candidates, one could investigate how the equilibrium choice of platforms by the candidates would be affected if it is known that a part of the voters are subject to the attraction effect. Similarly, industrial organization provides many settings in which one can examine the market-related consequences of the attraction effect. Indeed, a companion paper, Ok, Ortoleva, and Riella (2014), provides one such application in the context of monopolistic vertical product differentiation. That paper considers the standard screening problem of a monopolist who chooses price/quality bundles to offer to consumers whose quality valuation is private information—as in the classical model of Mussa and Rosen (1978)—but allows a fraction of the customers to be reference dependent as in the model characterized in Theorem 1. It is found that, under some parametric restrictions, the attraction effect would in equilibrium be exploited by the monopolist to (at least partly) overcome the incentive-compatibility constraints.

Appendix: Proofs

A. *Proof of Theorem* 1

We begin with the following definition:

DEFINITION A1: *Let* $\mathbf{c}$ *be a choice correspondence on* $\mathfrak{X}$. *For any given* $S \in \mathfrak{X}$, *we say that* S *is* $\mathbf{c}$-awkward *if there exist some doubleton subset* T *of* S *with* $\mathbf{c}(S) \cap T \neq \emptyset$ *and* $\mathbf{c}(S) \cap T \neq \mathbf{c}(T)$.

We can now make the following observation. Let $\mathbf{c}$ be a choice correspondence on $\mathfrak{X}$ that satisfies RCon, and take any $\mathbf{c}$-awkward $S \in \mathfrak{X}$. Let $\mathcal{T}_S$ stand for the collection of all T in $\mathfrak{X} \cap 2^S$ such that $\mathbf{c}(S) \cap T \neq \emptyset$ and $\mathbf{c}(T) \neq \mathbf{c}(S) \cap T$. As S is $\mathbf{c}$-awkward, there exists $T \in \mathcal{T}_S$ with $|T| = 2$. Besides, if $S = \cup \{T : T \in \mathcal{T}_S\}$, then $\mathcal{T}_S$ is a $\mathbf{c}$-cover of S, so by RCon, there is a $T \in \mathcal{T}$ with $\mathbf{c}(T) = \mathbf{c}(S) \cap T$, which is impossible in view of the definition of $\mathcal{T}_S$. It follows that $\cup \{T : T \in \mathcal{T}_S\}$ is a proper subset of S, that is, there is some $z \in S$ such that $\mathbf{c}(T) = \mathbf{c}(S) \cap T$ for all $T \in \mathfrak{X} \cap 2^S$ with $\mathbf{c}(S) \cap T \neq \emptyset$ and $z \in T$.

We have proved the "only if" part of the following characterization, whose "if" part is straightforward.

LEMMA A1: *Let* $\mathbf{c}$ *be a choice correspondence on* $\mathfrak{X}$. *Then,* $\mathbf{c}$ *satisfies RCon if and only if for any* $\mathbf{c}$*-awkward set* S *in* $\mathfrak{X}$, *there exists a* $z \in S$ *such that* $\mathbf{c}(T) = \mathbf{c}(S) \cap T$ *for all* $T \in \mathfrak{X} \cap 2^S$ *with* $\mathbf{c}(S) \cap T \neq \emptyset$ *and* $z \in T$.

We now move to the proof of Theorem 1.

Necessity.—Let $\mathbf{c}$ be a continuous choice correspondence on $\mathfrak{X}$ which is represented by a reference dependent choice model $\langle u, \mathbf{r}, \mathcal{U} \rangle$ on $\mathfrak{X}$. It readily follows from this representation that, for any $x, y \in X$, we have $x \in \mathbf{c}\{x, y\}$ if and only if $u(x) \geq u(y)$. It is also plain that $u(x) = u(y)$ for any $x, y \in X$ with $\{x, y\} \subseteq \mathbf{c}(S)$ for some $S \in \mathfrak{X}$. Thus $\mathbf{c}$ satisfies No-Cycle and RI. On the other hand, Proposition 2 entails that $\mathbf{c}$ satisfies RA. Finally, suppose S is a $\mathbf{c}$-awkward set in $\mathfrak{X}$. It is easy to see that this implies that $\mathbf{c}(S) \neq \arg\max u(S)$ so that $\mathbf{r}(S) \neq \Diamond$. Put $z := \mathbf{r}(S)$, and notice that (3) entails that $\mathbf{c}(T) = \mathbf{c}(S) \cap T$ for all $T \in \mathfrak{X} \cap 2^S$ with $\mathbf{c}(S) \cap T \neq \emptyset$ and $z \in T$. In view of the arbitrariness of S and Lemma A1, therefore, we may conclude that $\mathbf{c}$ satisfies RCon.

Sufficiency.—Suppose that $\mathbf{c}$ is a continuous choice correspondence on $\mathfrak{X}$ that satisfies the four axioms in the statement of the theorem. Define the binary relation $\succsim \subseteq X \times X$ by $x \succsim y$ if and only if $x \in \mathbf{c}\{x, y\}$. Since $\mathbf{c}$ is a continuous choice correspondence that satisfies No-Cycle, $\succsim$ is a complete and continuous preorder on X. Given that X is a separable metric space, therefore, we may invoke Debreu's Utility Representation Theorem to find a continuous function $u : X \to \mathbb{R}$ such that $x \succsim y$ if and only if $u(x) \geq u(y)$ for every $x, y \in X$. This implies $\mathbf{c}\{x, y\} = \arg\max u(\{x, y\})$ for every $x, y \in X$.

CLAIM A1: *A set* $S \in \mathfrak{X}$ *is not* $\mathbf{c}$*-awkward if and only if* $\mathbf{c}(S) = \arg\max u(S)$.

PROOF:

Assume $S \in \mathfrak{X}$ is not $\mathbf{c}$-awkward, and pick any $(x, y) \in \arg\max u(S) \times \mathbf{c}(S)$. Then, $x \in \mathbf{c}\{x, y\}$ and $y \in \mathbf{c}(S) \cap \{x, y\}$, while $\mathbf{c}\{x, y\} = \mathbf{c}(S) \cap \{x, y\}$ (because S is not $\mathbf{c}$-awkward). It follows that $x \in \mathbf{c}(S)$ and $y \in \arg\max u(S)$. Thus: $\mathbf{c}(S) = \arg\max u(S)$. The converse assertion follows readily from the definition of $\mathbf{c}$-awkwardness and the choice of u. ∎

CLAIM A2: *For each* $\mathbf{c}$*-awkward* $S \in \mathfrak{X}$, *there exists a* $z \in S$ *such that* z *is a revealed* $\mathbf{c}$*-reference for every element of* $\mathbf{c}(S)$ *and* $\mathbf{c}(T) = \mathbf{c}(S) \cap T$ *for all* $T \in \mathfrak{X} \cap 2^S$ *with* $\mathbf{c}(S) \cap T \neq \emptyset$ *and* $z \in T$.

PROOF:

Take any $\mathbf{c}$-awkward S in $\mathfrak{X}$. It follows from RI and the definition of u that $u(x) = u(w)$ for all $x, w \in \mathbf{c}(S)$. Thus, by Claim 1, there exists an alternative y in arg max $u(S) \backslash \mathbf{c}(S)$. By RCon and Lemma A1, we know that there is a $z \in S$ such that $\mathbf{c}(T) = \mathbf{c}(S) \cap T$ for all $T \in \mathfrak{X} \cap 2^S$ with $\mathbf{c}(S) \cap T \neq \emptyset$ and $z \in T$. This implies that y cannot belong to $\mathbf{c}\{x, y, z\}$ for any $x \in \mathbf{c}(S)$. Since $u(y) \geq u(x)$ for all $x \in \mathbf{c}(S)$, we conclude that z is a revealed $\mathbf{c}$-reference for every element of $\mathbf{c}(S)$. ∎

Define the Binary relation $R \subseteq X \times X$ by xRz if and only if $x = z$, or z is a revealed $\mathbf{c}$-reference for x. Let $\trianglerighteq$ be the transitive closure of R. By the definition of $\trianglerighteq$ and RA, for any $x, y \in X$, if z is a revealed $\mathbf{c}$-reference for x, then $x \trianglerighteq z$, and if $x \trianglerighteq z$, then z is a potential $\mathbf{c}$-reference for x.

Next, we define the map $\mathbf{r} : \mathfrak{X} \rightarrow X \cup \{\lozenge\}$ as follows: (i) If there exists $z \in S$ such that z is a revealed $\mathbf{c}$-reference for all $x \in \mathbf{c}(S)$ and $\mathbf{c}(T) = \mathbf{c}(S) \cap T$ for all $T \in \mathfrak{X} \cap 2^S$ with $\mathbf{c}(S) \cap T \neq \emptyset$ and $z \in T$, then let $\mathbf{r}(S)$ be any such z; and (ii) define $\mathbf{r}(S) := \lozenge$, otherwise. By the Axiom of Choice, $\mathbf{r}$ is well-defined. Moreover, by Claim A2, whenever $\mathbf{r}(S) = \lozenge$, S is not $\mathbf{c}$-awkward and, by Claim A1, $\mathbf{c}(S) =$ arg max $u(S)$. That is, we have the following claim:

CLAIM A3: *For any* $S \in \mathfrak{X}$, $\mathbf{c}(S) =$ arg max $u(S)$ *if* $\mathbf{r}(S) = \lozenge$.

Finally:

CLAIM A4: *If* $S \in \mathfrak{X}$ *and* $\mathbf{r}(S) \neq \lozenge$, *then* $\mathbf{c}(S) =$ arg max $u(S \cap \{\omega \in X : \omega \trianglerighteq \mathbf{r}(S)\}$.

PROOF:

Let $S \in \mathfrak{X}$ be such that $\mathbf{r}(S) \neq \lozenge$. By definition of $\mathbf{r}$ and $\trianglerighteq$, we have $\mathbf{r}(S) \in S$ and $\mathbf{c}(S) \subseteq \{\omega \in X : \omega \trianglerighteq \mathbf{r}(S)\}$. Pick any $x \in \mathbf{c}(S)$ and $y \in \{\omega \in X : \omega \trianglerighteq \mathbf{r}(S)\} \cap S$. Again, by definition of $\mathbf{r}$, we have $x \in \mathbf{c}\{x, y, \mathbf{r}(S)\} = \mathbf{c}(S) \cap \{x, y, \mathbf{r}(S)\}$. It then follows from the definition of $\trianglerighteq$ and RA that $\mathbf{r}(S)$ is a potential $\mathbf{c}$-reference for y, which implies that we cannot have $\{y\} = \mathbf{c}\{x, y\}$. That is, $u(x) \geq u(y)$, and we conclude that $\mathbf{c}(S) \subseteq$ arg max $u(S \cap \{\omega \in X : \omega \trianglerighteq \mathbf{r}(S)\})$. Now pick any $y \in$ arg max $u(S \cap \{\omega \in X : \omega \trianglerighteq \mathbf{r}(S)\})$ and $x \in \mathbf{c}(S)$. By the previous observation, we have $\{x, y\} = \mathbf{c}\{x, y\}$. Since $\mathbf{r}(S)$ is a potential $\mathbf{c}$-reference for y and $x \in \mathbf{c}\{x, y, \mathbf{r}(S)\}$, this implies $y \in \mathbf{c}\{x, y, \mathbf{r}(S)\}$. By definition of $\mathbf{r}$, therefore, $y \in \mathbf{c}(S)$. Conclusion: $\mathbf{c}(S) =$ arg max $u(S \cap \{\omega \in X : \omega \trianglerighteq \mathbf{r}(S)\})$. ∎

Because $\trianglerighteq$ is a preorder, it is well known that there exists a set $\mathcal{U}$ of real maps on X such that, for every x and y in X, $x \trianglerighteq y$ if and only if $U(x) \geq U(y)$ for every $U \in \mathcal{U}$ (cf. Evren and Ok 2011). It remains to show that u, $\mathbf{r}$, and $\mathcal{U}$ satisfy (3), but this is an easy consequence of the definitions of $\mathcal{U}$ and $\mathbf{r}$.

B. *Proof of Proposition 3*

For any given reference dependent choice model $\langle u, \mathbf{r}, \mathcal{U} \rangle$, (4) ensures that $\mathcal{R}$ depends only on the choice correspondence $\mathbf{c}$ that $\langle u, \mathbf{r}, \mathcal{U} \rangle$ represents. We next show that the same is true for the reference correspondence $\mathbf{R}$ associated with this model.

LEMMA A2: *Let* $\mathbf{c}$ *be a choice correspondence on* $\mathfrak{X}$ *that is represented by a reference dependent choice model* $\langle u, \mathbf{r}, \mathcal{U} \rangle$, *and let* $S \in \mathfrak{X}$ *be such that* $\mathbf{c}(S) \neq \arg\max u(S)$. *Then,* $z \in \mathbf{R}(S)$ *if and only if* (*i*) $z \in S$, (*ii*) z *is a revealed* $\mathbf{c}$*-reference for each* $x \in \mathbf{c}(S)$, *and* (*iii*) $\mathbf{c}(T) = \mathbf{c}(S) \cap T$ *for every* $T \in \mathfrak{X} \cap 2^S$ *with* $\mathbf{c}(S) \cap T \neq \emptyset$ *and* $z \in T$. (*In particular,* $\mathbf{c}$ *determines* $\mathbf{R}$ *completely, since Claim A*1 *above guarantees that, for any* $S \in \mathfrak{X}$, $\mathbf{c}(S) \neq \arg\max u(S)$ *if and only if S is* $\mathbf{c}$*-awkward.*)

PROOF:

Take any $z \in S$ that satisfies the properties (ii) and (iii). As S is $\mathbf{c}$-awkward, it follows from these properties and the construction of the reference dependent choice model given in the proof of Theorem 1 that there is a reference map $\mathbf{r}'$ on $\mathfrak{X}$ such that $\mathbf{r}'(S) = z$ and $\langle u, \mathbf{r}', \mathcal{U} \rangle$ also represents $\mathbf{c}$. Then, by (3), we have $\mathbf{c}(T) = \arg\max u(T \cap \mathcal{U}^{\uparrow}(z))$ for any $T \in \mathfrak{X} \cap 2^S$ with $\mathbf{c}(S) \cap T \neq \emptyset$ and $z \in T$. Conclusion: $z \in \mathbf{R}(S)$. Conversely, assume that z is an element of $\mathbf{R}(S)$. This implies that there exists a reference map $\mathbf{r}'$ on $\mathfrak{X}$ such that $\mathbf{r}'(S) = z$ and $\langle u, \mathbf{r}', \mathcal{U} \rangle$ also represents $\mathbf{c}$. Now Proposition 1 implies that z is a revealed $\mathbf{c}$-reference for each $x \in \mathbf{c}(S)$. It is also clear that z satisfies property (iii). Our lemma is thus proved. ∎

The following is a useful consequence of this observation.

LEMMA A3: *Let* $\mathbf{c}$ *be a choice correspondence on* $\mathfrak{X}$ *that is represented by a reference dependent choice model* $\langle u, \mathbf{r}, \mathcal{U} \rangle$. *Then, for any* $S \in \mathfrak{X}$ *with* $\mathbf{c}(S) \neq \arg\max u(S)$, *we have* $\mathbf{c}(S) = \arg\max u(S \cap \mathcal{R}(\mathbf{r}(S)))$.

PROOF:

This comes from the facts that, by Proposition 2, we have that $\mathcal{R}(\mathbf{r}(S)) \subseteq \mathcal{U}^{\uparrow}(\mathbf{r}(S))$ and, by Proposition 1, we have that $\mathbf{c}(S) \subseteq \mathcal{R}(\mathbf{r}(S))$. ∎

We now move to the proof of Proposition 3.

Necessity.—Suppose $\langle u_1, \mathbf{r}_1, \mathcal{U}_1 \rangle$ and $\langle u_2, \mathbf{r}_2, \mathcal{U}_2 \rangle$ represent the same choice correspondence. Then, u_1 and u_2 are continuous real functions on X that represent the same complete preorder on X (that arises from choices over pairwise choice problems), and hence, they must be continuous and strictly increasing transformations of each other. Furthermore, for each $i = 1, 2$, Lemma A2 and (4) ensure that $\mathbf{R}_i$ and $\mathcal{R}_i$ depend only on the choice correspondence that $\langle u_i, \mathbf{r}_i, \mathcal{U}_i \rangle$ represents. As these models represent the same choice correspondence, therefore, we must have $\mathbf{R}_1 = \mathbf{R}_2$ and $\mathcal{R}_1 = \mathcal{R}_2$.

Sufficiency.—Suppose that $\langle u_1, \mathbf{r}_1, \mathcal{U}_1 \rangle$ and $\langle u_2, \mathbf{r}_2, \mathcal{U}_2 \rangle$ are two reference dependent choice models on $\mathfrak{X}$ such that u_2 is a continuous and strictly increasing transformation of u_1, $\mathbf{R}_1 = \mathbf{R}_2$ and $\mathcal{R}_1 = \mathcal{R}_2$. Call the choice correspondences induced by these two models $\mathbf{c}_1$ and $\mathbf{c}_2$, respectively. Since $\mathbf{R}_1 = \mathbf{R}_2$, we know that, for every $S \in \mathfrak{X}$ with $\mathbf{c}_1(S) = \arg\max u_1(S)$, we have $\mathbf{c}_2(S) = \arg\max u_2(S) = \arg\max u_1(S) = \mathbf{c}_1(S)$. Consider now a choice problem S such that $\mathbf{c}_1(S) \neq \arg\max u_1(S)$. Since $\mathbf{r}_2(S) \in \mathbf{R}_2(S) = \mathbf{R}_1(S)$, we can find a reference map $\mathbf{r}'$ on $\mathfrak{X}$ such that $\mathbf{r}'(S) = \mathbf{r}_2(S)$ and $\langle u_1, \mathbf{r}_1, \mathcal{U}_1 \rangle$ and $\langle u_1, \mathbf{r}', \mathcal{U}_1 \rangle$ represent the same choice correspondence. By Lemma A3, this implies that $\mathbf{c}_1(S)$

$=$ arg max $u_1(S \cap \mathcal{R}_1(\mathbf{r}_2(S)))$ $=$ arg max $u_2(S \cap \mathcal{R}_2(\mathbf{r}_2(S)))$ $=$ $\mathbf{c}_2(S)$. We conclude that $\mathbf{c}_1(S) = \mathbf{c}_2(S)$, as we sought.

C. *Independence of the Axioms*

In this section we show the axioms that we imposed in Theorem 1 are independent.

Example A1: Let $X := \{x, y, z\}$, and consider the choice function $\mathbf{c}$ on $2^X \backslash \{\emptyset\}$ defined by $\mathbf{c}\{x, y\} := x$, $\mathbf{c}\{y, z\} := y$, $\mathbf{c}\{x, z\} := z$ and $\mathbf{c}(X) := x$. Clearly, $\mathbf{c}$ violates No-Cycle, but it satisfies RCon and RI. The only revealed $\mathbf{c}$-reference here is y, which is a revealed $\mathbf{c}$-reference for x. It follows that $\mathbf{c}$ satisfies RA.

Example A2: Let $X := \{x, y, z, w\}$, and consider the choice function $\mathbf{c}$ on $2^X \backslash \{\emptyset\}$ defined by $\mathbf{c}\{x, y\} = \mathbf{c}\{x, z\} = \mathbf{c}\{x, w\} = \mathbf{c}\{x, y, z\} = \mathbf{c}\{x, y, w\} = \mathbf{c}\{x, z, w\} := x$, $\mathbf{c}\{y, z\} = \mathbf{c}\{y, w\} = \mathbf{c}\{y, z, w\} := y$, and $\mathbf{c}\{z, w\} = \mathbf{c}(X) := z$. It is easy to see that $\mathbf{c}$ satisfies No-Cycle and RI. Also, there is no revealed $\mathbf{c}$-reference in this example, so $\mathbf{c}$ also satisfies RA. On the other hand, $\mathbf{c}$ violates RCon. For instance, $\mathcal{T} := \{\{x, z\}, \{y, z, w\}\}$ is a $\mathbf{c}$-cover of X, but we have $\mathbf{c}(X) \cap T \neq \mathbf{c}(T)$ for each $T \in \mathcal{T}$.

Example A3: Let $X := \{x, y, z, w\}$, and consider the choice function $\mathbf{c}$ on $2^X \backslash \{\emptyset\}$ defined by $\mathbf{c}\{x, y\} = \mathbf{c}\{x, z\} = \mathbf{c}\{x, w\} = \mathbf{c}\{x, y, z\} := x$, $\mathbf{c}\{y, z\} = \mathbf{c}\{y, w\} = \mathbf{c}\{x, y, w\} := y$, $\mathbf{c}\{z, w\} = \mathbf{c}\{x, z, w\} = \mathbf{c}\{y, z, w\} := z$ and $\mathbf{c}(X) := z$. It is easy to check that $\mathbf{c}$ satisfies No-Cycle, RCon, and RI. On the other hand, w is a revealed $\mathbf{c}$-reference for y, but w is not a potential $\mathbf{c}$-reference for y (because $\mathbf{c}\{y, z\} = y$ and yet $\mathbf{c}\{y, z, w\} = z$). Thus, $\mathbf{c}$ violates RA.

Example A4: Let $X := \{x, y, z\}$, and consider the choice correspondence $\mathbf{c}$ on $2^X \backslash \{\emptyset\}$ defined by $\mathbf{c}\{x, y\} = \mathbf{c}\{x, z\} := \{x\}$, $\mathbf{c}\{y, z\} := \{y\}$, and $\mathbf{c}(X) := \{x, y\}$. It is plain that $\mathbf{c}$ satisfies No-Cycle but not RI. On the other hand, the only revealed $\mathbf{c}$-reference here is z which is a revealed $\mathbf{c}$-reference for y. It follows that $\mathbf{c}$ satisfies RA. It is also easily checked that $\mathbf{c}$ satisfies RCon.

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